FINDING THE VELOCITY OF LAMINAR FLUID FLOW

Prishchenko O. P., Chernogor T. T.
National Technical University «Kharkiv Polytechnic Institute», Kharkiv

Newton's basic law of viscous flow, is as follows:

$$F = \eta \frac{v_2 - v_1}{r_2 - r_1} S \qquad (1)$$

where F is the internal friction force; S is the layer area; v_1 and v_2 are the velocities of the layers, respectively, lagging behind the tube wall at distance r_1 and r_2; η is the viscosity coefficient.

Let the length of the tube be l and the pressure at its inlet and outlet holes be p and p_0, respectively. Based on the basic law of viscous flow (1), we obtain an expression for the flow velocity in differential form. For this purpose, let us select an elementary cylinder in the tube with length Δl and radius r by the value Δr (see Fig. 1). This cylinder will experience friction, the force of which, according to the law (1), will amount to $\Delta F = \eta \frac{\Delta y}{\Delta r} \Delta S,$ where ΔS is the side surface area of the cylinder. Therefore $\Delta F = \eta \frac{\Delta y}{\Delta r} 2\pi r \Delta l$. The friction force is counteracted by the pressure force, which is equal to ΔF and opposite to it in direction, i.e., $\Delta F = -\pi r^2 \Delta P,$ $\pi r^2 -$ base area of the elementary cylinder; $P = p - p_0$

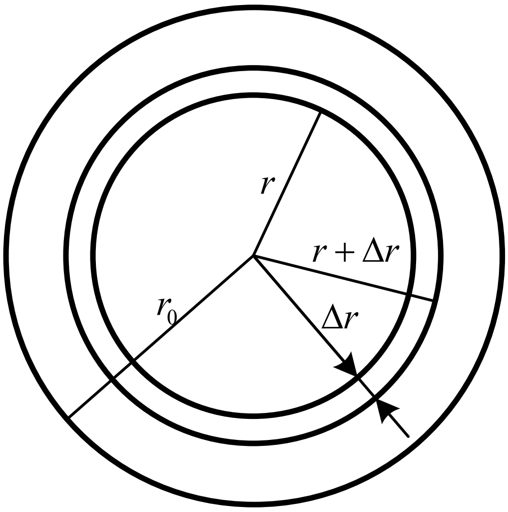

Fig. 1

Equating the first parts of the last two expressions of the increment ΔF, we obtain $\Delta v = -\frac{1}{2\eta} \frac{\Delta P}{\Delta l} r \Delta r.$ Since the cross sections of the tube are the same along its entire length, the pressure in the tube is distributed uniformly and is equal to $\frac{p - p_0}{l}$. Therefore $\frac{\Delta P}{\Delta l} = \frac{p - p_0}{l}$. So, uh., $\Delta v = -\frac{1}{2\eta} \frac{p - p_0}{l} r \Delta r,$ or in differential form $dv = -\frac{1}{2\eta} \frac{p - p_0}{l} r dr.$ The simplest differential equation is obtained.

Let us find the laminar flow velocity as a function of r if the radius of the tube is r_0 and $v(r_0) = 0$. Integrating the last differential equation under the condition that $v(r_0) = 0$, we obtain $v = -\frac{p - p_0}{2\eta} \int_{r_0}^{r} \rho d\rho,$ or finally

$$v = \frac{1}{4\eta} \frac{p - p_0}{l} \left(r_o^2 - r^2 \right).$$