SPECTRAL DECOMPOSITIONS OF RANDOM SEQUENCES OF AN INFINITE RANK OF NON-STATIONARY

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The paper deals with the problem of spectral decomposition of an infinitive rank of non-stationary.

Using difference and functional equations associated with operator knot, we construct the spectral decompositions of random sequences of an infinite rank of non-stationarity, which are considered as sequences in the relevant Hilbert space H .

Theorem. When $x(n)=A^n x_0\ (n\geq 0)$($A-$ linear bounded operator in Hilbert space $H,\ x_0\in H$) complete dissipative sequence of finite quasi-rank, then for each n there are two sequences $z_k(n)$ and $u_k(n)$: $z_k(n)=\psi_k(n)z_k$, $Mz_k\overline{z_j}=\delta_{kj}$, where δ_{kj} $-$Kronecker symbol.

$\psi_k(n)$ $-$a determinized sequence for each n, and there is a presentation

$$x_n=\sum_{k=1}^{\infty}z_k(n)=\sum_{k=1}^{\infty}\psi_k(n)z_k .$$

Sequences $u_k(n)$ are canal:

$$u_k(n)=\sum_{\alpha=1}^{N}u_k^{(\alpha)}(n)a_\alpha .$$

At that, $u_k^{(\alpha)}(n)=\left\langle u_k(n),a_\alpha\right\rangle_E=Mu_k(n)\overline{a_\alpha},\quad Ma_\alpha\overline{a_\beta}=\left\langle a_\alpha,a_\beta\right\rangle_E=\delta_{\alpha\beta}$, where $\delta_{\alpha\beta}$ $-$ Kronecker symbol.

The functions of the discrete argument $\psi_k(n)=u_k^{(\alpha)}(n)$ are determined from the system of the first order recurrent difference equations:

$$i\psi_k(n+1)+\lambda_k\psi_k(n)=\sum_{\alpha=1}^{r}u_k^{(\alpha)}(n)\sqrt{\omega_\alpha}Ma_\alpha\overline{z_k}u_k^{(\alpha)}(n);$$

$$\psi_k(n)\Big|_{n=0}=\psi_k(0);$$

$$u_{k+1}^{(\alpha)}(n)=u_k^{(\alpha)}(n)-i\sqrt{\omega_\alpha}Ma_\alpha\overline{z_k}\psi_k(n);$$

$$u_k^{(\alpha)}(n)\Big|_{k=0}=0,\quad u_k^{(\alpha)}(n)=Mu_k(n)\overline{a_\alpha},$$

where λ_k – operator's own numbers A, a ω_α–operator's own numbers $2\operatorname{Im}A$.