FORMATION OF THE SOLUTION SEARCH SPACE IN THE PROBLEM OF FINDING A HAMILTONIAN CYCLE ON A GRAPH

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The complexity of the problem of finding a Hamiltonian cycle on a graph of ***n*** vertices is ***(n-1)!*** that's how much it takes to iterate over the options to find the best solution. This problem is used in various practical applications, is related to ***NP*** class of complexity, but interest in it remains.

For the algorithm of solving the problem the requirements are presented: complexity – not higher than polynomial; accuracy-guaranteeing the optimal solution; universality-applicability for any graph. There is no algorithm for solving this problem that meets these requirements due to the inability to formulate conditions that determine the optimal solution of the problem.

Therefore, the main way to solve the problem is to enumerate possible solutions to choose the best one. But to sort through all the space of solutions is simply not real. So the only possible solution is to reduce the search space. For the solution, a new method is proposed, which significantly reduces the search.

The complete space of possible solutions of the problem is the set of all cycles that can be constructed for the selected initial vertex, and any vertex of the graph can be initial. The optimal solution is a Hamiltonian cycle of minimum (shortest) length. It must contain in its structure shortest paths between some (or even all) vertices of the graph. Therefore, solutions that do not contain the shortest path between any pair of vertices of the graph should not get into the search space. On the contrary, solutions that contain such shortest paths should get into the search space.

But moving from the initial vertex along the shortest path, we can simply not build a Hamiltonian cycle. This can be seen if you try to solve this problem by simply using the algorithm to find the shortest path in the graph from a given initial vertex.

From each vertex of the graph, there can be ***(n-1)*** shortest paths to the remaining vertices. Each of these paths can be tried to complete to the Hamiltonian cycle. And, of course, the cycle obtained in each case will be shorter if the completed part contains shortest paths between some vertices of the graph. This means that the construction of possible solutions for the search space should be based on the shortest paths between vertices in the graph.

Following this logic, the first step is to construct all shortest paths between each pair of vertices in the graph and at the second stage to form possible solutions from these shortest paths, which will make up the search space. The best solution from this space will be the optimal solution.

The proposed method can reduce the search space to size $n^2 – n$ and implement an algorithm that satisfies the formulated requirements.