Floquet scattering theory for current and heat noise in large amplitude adiabatic pumps

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We discuss the statistical correlation properties of currents and energy flows generated by an adiabatic quantum pump. Our approach emphasizes the important role of quantized energy exchange between the sea of electrons and the oscillating scatterer. The frequency ω of oscillations introduces a natural energy scale $\hbar\omega$. In the low temperature limit $k_BT \ll \hbar\omega$ the pump generates a shot-like noise which manifests itself in photon-assisted quantum mechanical exchange amplitudes. In the high temperature limit $k_BT \gg \hbar\omega$ the pump produces a thermal-like noise due to ac-currents generated by the pump. We predict that with increasing temperature the frequency dependence of the noise changes. The current noise is linear in ω at low temperatures, is quadratic at intermediate temperatures, and is linear again at high temperatures. Similarly, in the same temperature regions, the heat flow noise is proportional to ω^3, ω^2, and ω.

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I. INTRODUCTION

Quantum pumping, a phenomenon in which a periodic local perturbation gives rise to a directed current in a phase coherent mesoscopic system, attracts great attention of both experimentalists[1−4] and theorists[5−33]. The physical mechanism leading to adiabatic quantum pumping involves quantum-mechanical interference and dynamical breaking of time-reversal invariance. This mechanism is relevant not only for open (i.e., connected to external particle reservoirs) systems but also for closed (ring-like) mesoscopic systems.[34,35,36]

The possibility[5,14,37−44] to achieve quantized transport, not only of charge but in addition heat[14,22,45,46] and spin[3,47−60] currents, makes pumping interesting also in view of possible applications. Quantum pumping has been investigated in systems of strongly correlated electrons[8,47,55,61], for systems in the quantum Hall regime[62,63] and in hybrid superconducting-normal structures.[64−70] Clearly any mesoscopic system with periodically evolving properties is able to exhibit a quantum pump effect.

Since the pump works under the influence of a time-dependent (periodic) perturbation it generates time-dependent (ac) currents.[6,71] The dc current, which is mainly of interest, is just the time averaged fully time-dependent current generated by the pump. The ac currents manifest themselves, for instance, in an interference effect[71] with ac currents driven through the pump if an external ac bias[4,72−74] is applied. In addition, as we will show in this work, the ac currents are visible in the noise of an unbiased pump, Fig.1.

The noise of a pump is important because it is closely related to whether quantized pumping is possible.[14,75,76] In addition the noise contains information about the physical processes taking place in quantum pumps which can not be obtained by considering only the time-averaged pump current.[26,45,77−81]

In the present paper we use the Floquet scattering matrix approach[82−87] to investigate the quantum statistical correlation properties (noise) of multi-terminal adiabatic quantum pumps, Fig.1. Our approach is based on the scattering matrix approach to ac transport in phase coherent mesoscopic systems[6]. According to this approach the currents flowing in the system are determined by the scattering of electrons com-

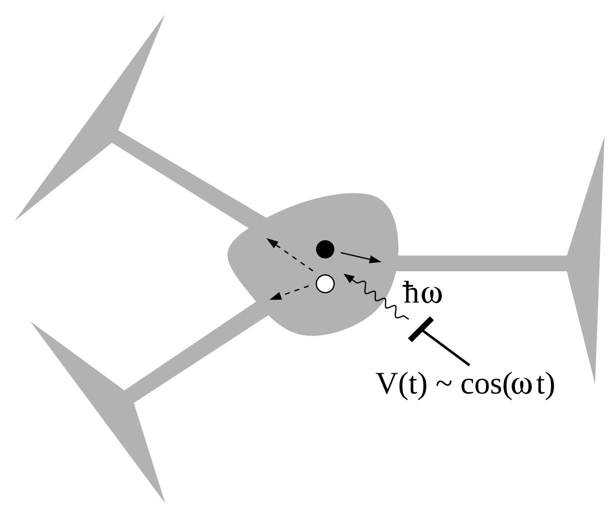

FIG. 1: Noise of a quantum pump: Two particles, an electron (open circle) and a hole (black circle), are involved in the scattering process relevant for the quantum noise. Different final states of a particle are possible: a particle can be transmitted into either of the leads (shown by dashed lines). These processes are described by photon-assisted quantum-mechanical exchange amplitudes.

ing from the reservoirs by the mesoscopic sample[88,89]. The basics of the approach used here is presented in Refs. 22,71 and the results obtained here generalize Ref. 45 to the case of a large amplitude pump.

Starting from a general formalism we mainly deal with the low frequency limit that differentiates our work from other works employing the Floquet approach to a current noise problem and concentrating on a limit of high driving frequencies (see, e.g., Refs. 80,81). The low frequency limit corresponds to *adiabatic* quantum pumping which was investigated experimentally in Ref. 1.

The Floquet scattering matrix approach, on the one hand, emphasizes the existence of side bands of electrons exiting the pump: these side bands are directly connected to the ac currents generated by the pump. On the other hand, this approach allows to reformulate an intrinsically non-stationary problem in terms of a stationary one. The latter is more convenient for the investigation reported here, for instance, it permits a transparent discussion of heat flow and its fluctuations. Therefore within the framework of the Floquet scattering matrix approach we can consider two aspects of particle flow fluctuations on the same footing: (i) fluctuations of the charge (current) flow, and (ii) fluctuations of the energy (heat) flow. We investigate these fluctuations and their dependence on the pump frequency at low and at relatively high temperatures.

The paper is organized as follows. In Sec.II we derive the general expressions for the current and heat flow noise in terms of the Floquet scattering matrix elements, and outline the adiabatic approximation used in the following sections. In Secs.III and IV we calculate the current noise and the heat flow noise, respectively, generated by an adiabatic quantum pump. In Sec.V we apply the general expressions of previous sections to a pump consisting of two oscillating delta-function barriers. We conclude in Sec.VI.

II. FLOQUET SCATTERING MATRIX EXPRESSIONS FOR NOISE

We consider a mesoscopic sample (scatterer) connected to N_r reservoirs via single channel leads. The reservoirs are assumed to to be at equal temperatures T_α and electrochemical potentials μ_α

$$\mu_\alpha = \mu; \quad T_\alpha = T, \quad \alpha = 1, \dots, N_r. \tag{1}$$

Let $\hat{a}_\alpha(E)$ and $\hat{b}_\alpha(E)$ be the annihilation operators for particles incident from and outgoing to the reservoir α, respectively. The operators $\hat{a}_\alpha(E)$ for incoming particles obey the following anti commutation relation

$$[\hat{a}^\dagger_\alpha(E), \hat{a}_\beta(E')] = \delta_{\alpha\beta}\delta(E - E'). \tag{2}$$

We assume that all the reservoirs are unaffected by the coupling to the scatterer and are in the equilibrium state.

Consequently, in all the leads, the quantum statistical average denoted by $\langle \dots \rangle$ of the incoming particles are those of an electron system at equilibrium. In particular, $\langle \hat{a}^\dagger_\alpha(E)\hat{a}_\beta(E')\rangle = \delta_{\alpha\beta}\delta(E - E')f_0(E)$ is proportional to the equilibrium Fermi distribution function,

$$f_0(E) = \frac{1}{\exp\left(\frac{E-\mu}{k_B T}\right) + 1}. \tag{3}$$

k_B is the Boltzmann constant.

The operators $\hat{b}_\alpha(E)$ for outgoing particles are related to the operators $\hat{a}_\alpha(E)$ through the scattering matrix of the sample under consideration.[89]

We consider a sample that is subject to external forces that are periodic in time. In particular its scattering properties oscillate in time with period $\mathcal{T} = 2\pi/\omega$. As a consequence an electron interacting with a scatterer can gain or loss one or several energy quanta $n\hbar\omega,\ n = 0, \pm1, \pm2, \dots$. Scattering on such an oscillatory scatterer can be described via the Floquet scattering matrix. We are interested in the sub-matrix of the Floquet matrix which describes the transitions between the propagating states.[22] We denote this sub-matrix by $\hat{S}_F$. The elements $S_{F,\alpha\beta}(E_n, E)$ of this matrix are the quantum mechanical amplitudes for an electron with energy E entering the scatterer through lead β and to leave the scatterer with energy $E_n = E + n\hbar\omega$ through lead α. The particle has thus absorbed ($n > 0$) or emitted ($n < 0$) energy quanta $|n|\hbar\omega$. The matrix $\hat{S}_F$ is unitary:

$$\sum_\alpha \sum_{n=-\infty}^{\infty} S^*_{F,\alpha\beta}(E_n, E) S_{F,\alpha\gamma}(E_n, E_m) = \delta_{m0}\delta_{\beta\gamma}, \tag{4a}$$

$$\sum_\beta \sum_{n=-\infty}^{\infty} S^*_{F,\alpha\beta}(E, E_n) S_{F,\gamma\beta}(E_m, E_n) = \delta_{m0}\delta_{\alpha\gamma}. \tag{4b}$$

Using the Floquet scattering matrix we can express the annihilation operators for outgoing particles in terms of the annihilation operators of the incoming particles,

$$\hat{b}_\alpha(E) = \sum_\beta \sum_n S_{F,\alpha\beta}(E, E_n)\hat{a}_\beta(E_n). \tag{5}$$

The unitarity conditions Eqs. (4) guarantee that the operators for outgoing particles obey the same anti-commutation relations as operators for the incoming particles.

A. Current noise

The correlation function $\mathbf{P}_{\alpha\beta}(t_1, t_2)$ of currents is[90] :

$$\mathbf{P}_{\alpha\beta}(t_1, t_2) = \frac{1}{2}\langle \Delta\hat{I}_\alpha(t_1)\Delta\hat{I}_\beta(t_2) + \Delta\hat{I}_\beta(t_2)\Delta\hat{I}_\alpha(t_1)\rangle, \tag{6}$$

Here $\hat{I}_\alpha(t)$ is the current operator in lead α and $\Delta\hat{I}_\alpha(t) = \hat{I}_\alpha(t) - \langle \hat{I}_\alpha(t)\rangle$. In the low temperature and low frequency limit of interest here

$$k_B T,\ \hbar\omega \ll \mu, \tag{7}$$

the operator $\hat{I}_\alpha(t)$ is[89]

$$\hat{I}_\alpha(t) = \frac{e}{h}\int dE dE' [\hat{b}^\dagger_\alpha(E)\hat{b}_\alpha(E') - \hat{a}^\dagger_\alpha(E)\hat{a}_\alpha(E')] e^{i\frac{E-E'}{\hbar}t}. \tag{8}$$

In the present work we investigate the correlation function for currents generated by the oscillating scatterer (a quantum pump). Since the pump is non-stationary the function $\mathbf{P}_{\alpha\beta}(t_1, t_2)$ depends on two times. However the pump works slowly: the pump frequency is small compared to the Fermi energy ($\omega \ll \mu/\hbar$) and it generates only noise in states lying close to the zero frequency region. To make this noise visible we need to eliminate the contribution coming from high frequency quantum fluctuations. To this end we integrate the correlation function $\mathbf{P}_{\alpha\beta}(t_1, t_2)$ over the time difference $\tau = t_1 - t_2$ and get the function $\mathbf{P}_{\alpha\beta}(t)$ depending only on the middle time $t = (t_1 + t_2)/2$. This function is periodic in time with period $\mathcal{T}$ and it is a correlation function of the currents generated by the pump.[71]

We are interested in the zero-frequency Fourier coefficient of this function. Thus the quantity of interest is:

$$\mathbf{P}_{\alpha\beta} = 2\int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \int_{-\infty}^{\infty} d\tau \mathbf{P}_{\alpha\beta}\Big(t+\frac{\tau}{2}, t-\frac{\tau}{2}\Big) \tag{9}$$

The zero-frequency noise considered here corresponds to a noise measurement which long compared to all intrinsic frequencies and long compared to the pump frequency. We remark that for very slow pumping cycles one could alternatively investigate the noise spectrum for a particular value of the pump parameters, assuming that the noise measurement is carried out on a times-scale short compared to the pump frequency.

Using Eqs. (5), (8) we can express $\mathbf{P}_{\alpha\beta}$ in terms of the Floquet scattering matrix:

$$\mathbf{P}_{\alpha\beta} = \frac{2e^2}{h}\int_0^{\infty} dE\Big(\mathbf{P}^{(th)}_{\alpha\beta}(E) + \mathbf{P}^{(sh)}_{\alpha\beta}(E)\Big), \tag{10a}$$

$$\mathbf{P}^{(th)}_{\alpha\beta}(E) = f_0(E)[1-f_0(E)]\Big(\delta_{\alpha\beta} + \sum_{n=-\infty}^{\infty}\Big\{\delta_{\alpha\beta}\sum_{\gamma}\big|S_{F,\alpha\gamma}(E_n,E)\big|^2 - \big|S_{F,\alpha\beta}(E_n,E)\big|^2 - \big|S_{F,\beta\alpha}(E_n,E)\big|^2\Big\}\Big). \tag{10b}$$

$$\mathbf{P}^{(sh)}_{\alpha\beta}(E) = \sum_{\gamma,\delta}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\frac{[f_0(E_n)-f_0(E_m)]^2}{2} S^*_{F,\alpha\gamma}(E,E_n)S_{F,\alpha\delta}(E,E_m)S^*_{F,\beta\delta}(E_p,E_m)S_{F,\beta\gamma}(E_p,E_n). \tag{10c}$$

The first term $\mathbf{P}^{(th)}_{\alpha\beta}$ is thermal (or Nyquist - Johnson) noise modified by the pump. It is due to thermal fluctuations of the incident states and disappears at zero temperature $T = 0$.

The second term $\mathbf{P}^{(sh)}_{\alpha\beta}$ is a shot noise contribution. It is entirely due to a working pump. The terms in Eq. (10c) describe quantum mechanical exchange during scattering of particles from the leads γ and δ to the leads α and β. The only difference from the stationary case is that during a scattering process an electron can change its energy absorbing or emitting energy quanta $\hbar\omega$. The quanta emitted/absorbed are counted by n, m and p in Eqs. 10b 10c. In the scattering processes which contribute to the noise two particles with energy E_n and E_m are incident through leads γ and δ, respectively. The particles leave the scatterer through the lead α having energy E and through the lead β having energy $E_p = E + p\hbar\omega$.

The term $\mathbf{P}^{(sh)}_{\alpha\beta}$ contributes both at finite as well as at zero temperature. However it vanishes if the pump does not work. This is so, since for a stationary scatterer only energy conserving transitions are allowed. In Eq. (10c), for the stationary scatterer, only the terms with $n = m = p = 0$ remain. For such terms the difference $f_0(E_n) - f_0(E_m)$ is identically zero.

Using the unitarity conditions Eqs. (4) we can check that the above noise power is subject to the conservation laws:[89]

$$\sum_{\beta}\mathbf{P}_{\alpha\beta} = 0, \quad \sum_{\alpha}\mathbf{P}_{\alpha\beta} = 0. \tag{11}$$

To prove the second equality one needs to make the shifts $E \to E - p\hbar\omega$, $n \to n - p$, and $m \to m - p$ in Eq. (10c).

Let us consider the sign of current correlations. The auto correlations ($\alpha = \beta$) are positive and the cross correlations ($\alpha \neq \beta$) are negative as it should be for fermions.[89,90] The thermal noise $\mathbf{P}^{(th)}_{\alpha\beta}(E)$ satisfies obviously this sign rule. To show this for the shot noise contribution we follow Ref. 89 and rewrite Eq. (10c):

$$\mathbf{P}^{(sh)}_{\alpha\beta}(E) = -\sum_{p=-\infty}^{\infty}\bigg|\sum_{n=-\infty}^{\infty}\sum_{\gamma} f_0(E_n)S^*_{F,\alpha\gamma}(E,E_n)S_{F,\beta\gamma}(E_p,E_n)\bigg|^2 < 0, \quad \alpha \neq \beta. \tag{12}$$

Since the cross correlations are negative and because of the sum rule Eq. (11) we conclude that the auto correlations are necessarily positive $\mathbf{P}^{(sh)}_{\alpha\alpha}(E) > 0$.

We remark that cross-correlations can be positive even for normal conductors if they are imbedded in an external circuit with a finite impedance, and in particular even if they are subject to oscillating voltages applied to the terminals of the conductor. In addition interactions can change the sign of correlations. We refer the interested reader to recent discussions[91,92]. However, none of these factors plays a role here and the cross-correlations are negative as in a sample subject to dc-transport and connected to an ideal zero-impedance external circuit.

Strictly speaking one can not rigorously separate the shot noise from the thermal noise especially at $\hbar\omega \sim k_BT$. Here we have introduced the two contributions, $\mathbf{P}^{(th)}_{\alpha\beta}$ and $\mathbf{P}^{(sh)}_{\alpha\beta}$, for convenience only. Their distinction is insofar justified as they depend differently on the temperature and pump frequency.

B. Heat flow fluctuations

By analogy with the current operator $\hat{I}_\alpha$ Eq. (8) we introduce a heat flow operator in the lead α :

$$\begin{gathered}\hat{I}_{E,\alpha}(t) = \frac{1}{h}\int dE dE'(E-\mu) \\ \times[\hat{b}^\dagger_\alpha(E)\hat{b}_\alpha(E') - \hat{a}^\dagger_\alpha(E)\hat{a}_\alpha(E')]e^{i\frac{E-E'}{\hbar}t}.\end{gathered} \tag{13}$$

The heat flow $\hat{I}_{E,\alpha}$ is the difference between the flows of energy (measured with respect to the Fermi energy μ) carried by electrons from the scatterer to the reservoir α and in the opposite direction. This definition makes sense since the operators $(\hat{b}^\dagger_\alpha(E), \hat{a}^\dagger_\alpha(E), \dots)$ correspond to particles with a definite energy E.

The correlation function $\mathbf{P}_{E,\alpha\beta}(t_1,t_2)$ of heat flows and the zero frequency heat flow noise power $\mathbf{P}_{E,\alpha\beta}$ of interest here is defined in a full analogy with Eqs. (6) and (9), [replacing $\hat{I}_\alpha(t)$ by $\hat{I}_{E,\alpha}(t)$].

Using Eq. (5) we express the zero frequency heat flow noise power $\mathbf{P}_{E,\alpha\beta}$ in terms of the Floquet scattering matrix as follows:

$$\mathbf{P}_{E,\alpha\beta} = \frac{2}{h}\int\limits_0^\infty dE\Big(\mathbf{P}^{(th)}_{E,\alpha\beta}(E) + \mathbf{P}^{(sh)}_{E,\alpha\beta}(E)\Big), \tag{14a}$$

$$\begin{gathered}\mathbf{P}^{(th)}_{E,\alpha\beta}(E) = k_BT\left(-\frac{\partial f_0(E)}{\partial E}\right)\Bigg(\delta_{\alpha\beta}(E-\mu)^2 + \sum_{n=-\infty}^{\infty}\Big\{\delta_{\alpha\beta}(E_n-\mu)^2\sum_\gamma\left|S_{F,\alpha\gamma}(E_n,E)\right|^2 \\ -(E-\mu)(E_n-\mu)\Big[\left|S_{F,\alpha\beta}(E_n,E)\right|^2 + \left|S_{F,\beta\alpha}(E_n,E)\right|^2\Big]\Big\}\Bigg).\end{gathered} \tag{14b}$$

$$\mathbf{P}^{(sh)}_{E,\alpha\beta}(E) = \sum_{\gamma,\delta}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}(E-\mu)(E_p-\mu)\frac{[f_0(E_n)-f_0(E_m)]^2}{2}S^*_{F,\alpha\gamma}(E,E_n)S_{F,\alpha\delta}(E,E_m)S^*_{F,\beta\delta}(E_p,E_m)S_{F,\beta\gamma}(E_p,E_n). \tag{14c}$$

In analogy with the current noise power we divide the whole heat flow noise power into two parts. $\mathbf{P}^{(th)}_{E,\alpha\beta}$ is termed the thermal heat noise since it disappears at zero temperature. $\mathbf{P}^{(sh)}_{E,\alpha\beta}(E)$ is called the heat flow shot noise.

Since the pump is a source of energy flows from the scatterer to the reservoirs for the heat flow noise power $\mathbf{P}_{E,\alpha\beta}$ we do not expect a conservation law like Eq. (11).

C. Adiabatic approximation

To calculate the noise power $\mathbf{P}_{\alpha\beta}$ one needs to know the Floquet scattering matrix. That requires the solution of a full time-dependent scattering problem. However for a slowly (adiabatically) oscillating scatterer the solution of a stationary scattering problem is sufficient.[22,71]

Let us assume that the scattering of electrons with energy E by the stationary sample can be described via the (stationary) scattering matrix $\hat{S}(E,\{x\})$ depending on some parameters $x_i \in \{X\}, i = 1,2,\dots,N_p$ (e.g., sample's shape, the strength of coupling to leads, etc.). Varying these parameters one can change the scattering properties of a sample. We suppose these parameters to be periodic functions in time: $x_i(t) = x_i(t+\mathcal{T}), \forall i$. Then the matrix $\hat{S}$ becomes dependent on time: $\hat{S}(E,t) = \hat{S}(E,\{x(t)\})$. In general the matrix $\hat{S}(E,t)$ does not describe the scattering of electrons by the time-dependent scatterer: only the Floquet scattering matrix $\hat{S}_F(E_n,E)$ does.

However if the parameters x_i change so slowly that the energy quantum $\hbar\omega$ is small compared with the relevant energy scale δE over which the scattering matrix $\hat{S}(E)$ changes significantly

$$\hbar\omega \ll \delta E \ll \mu, \tag{15}$$

and therefore $\hat{S}(E) \approx \hat{S}(E+\hbar\omega)$, then the elements of the Floquet scattering matrix can be expressed in terms of the Fourier coefficients of the matrix $\hat{S}(t)$. Note the energy scale δE can vary from case to case. For instance, close to a transmission resonance δE is a resonance width but far from the resonance δE is rather the distance between resonances. For many channel scatterers well connected to leads δE is the Thouless energy.

The first two terms of an expansion of the Floquet scattering matrix in powers of $\hbar\omega/\delta E \ll 1$ read[71]

$$\hat{S}_F(E_n,E) = \hat{S}_n\left(\frac{E_n+E}{2}\right) + \hbar\omega\hat{A}_n(E) + O\big([\hbar\omega/\delta E]^2\big). \tag{16}$$

Here the matrix of Fourier coefficients is defined in the following way ($Y \equiv A, S$):

$$\begin{gathered}\hat{Y}_n(E) = \frac{\omega}{2\pi}\int\limits_0^{\mathcal{T}} dt e^{in\omega t}\hat{Y}(E,t), \\ \hat{Y}(E,t) = \sum_{n=-\infty}^{\infty} e^{-in\omega t}\hat{Y}_n(E).\end{gathered} \tag{17}$$

The matrix $\hat{A}$ is subject to the following equation

$$\hbar\omega\left(\hat{S}^\dagger(E,t)\hat{A}(E,t) + \hat{A}^\dagger(E,t)\hat{S}(E,t)\right) = \frac{1}{2}\mathcal{P}\{\hat{S}^\dagger;\hat{S}\}, \tag{18a}$$

$$\mathcal{P}\{\hat{S}^\dagger;\hat{S}\} = i\hbar\left(\frac{\partial\hat{S}^\dagger}{\partial t}\frac{\partial\hat{S}}{\partial E} - \frac{\partial\hat{S}^\dagger}{\partial E}\frac{\partial\hat{S}}{\partial t}\right). \tag{18b}$$

Note the matrix $\mathcal{P}\{\hat{S}^\dagger;\hat{S}\}$ is traceless.

Without magnetic fields when the stationary scattering matrix is symmetric in lead indices, $S_{\alpha\beta} = S_{\beta\alpha}$, the matrix $\hat{A}$ is antisymmetric:[71] $A_{\alpha\beta} = -A_{\beta\alpha}$.

III. ADIABATIC CURRENT NOISE POWER

In this section we use the adiabatic approximation Eqs. (16), (18) to calculate the current noise power Eq. (10). A working pump modifies the thermal noise according to Eq. (10b) and it is a source of shot noise Eq. (10c).

A. Current thermal noise

In the high temperature regime, relevant for existing experiments,

$$\hbar\omega \ll k_B T, \tag{19}$$

the main source of noise are equilibrium thermal fluctuations. A working pump modifies the thermal noise only slightly. However the noise produced by the pump has a characteristic dependence on frequency and temperature and thus can be detected.

To calculate the thermal noise $\mathbf{P}^{(th)}_{\alpha\beta}$ in the presence of a slowly oscillating scatterer we substitute the adiabatic expansion Eqs. (16) for the Floquet scattering matrix into Eq. (10b). Since we find the Floquet scattering matrix with an accuracy of ω we calculate the noise with the same accuracy.

After a little algebra (see Appendix, Sec. 1) we find for the zero-frequency thermal noise:

$$\mathbf{P}^{(th)}_{\alpha\beta} = \frac{2e^2}{h}\int\limits_0^\infty dE\Big(\mathbf{P}^{(th,0)}_{\alpha\beta}(E) + \mathbf{P}^{(th,p)}_{\alpha\beta}(E)\Big), \tag{20a}$$

$$\mathbf{P}^{(th,0)}_{\alpha\beta}(E) = k_B T\left(-\frac{\partial f_0}{\partial E}\right)\left(2\delta_{\alpha\beta} - \overline{\left|S_{\alpha\beta}(E)\right|^2} - \overline{\left|S_{\beta\alpha}(E)\right|^2}\right), \tag{20b}$$

$$\mathbf{P}^{(th,p)}_{\alpha\beta}(E) = k_B T\left(-\frac{\partial f_0}{\partial E}\right)\frac{h}{e}\left(\delta_{\alpha\beta}\frac{dI_\alpha(E)}{dE} - \frac{dI^{(s)}_{\alpha\beta}(E)}{dE}\right). \tag{20c}$$

The first term $\mathbf{P}^{(th,0)}_{\alpha\beta}$ is the equilibrium (Nyquist-Johnson) noise in the presence of a working pump. The second term $\mathbf{P}^{(th,p)}_{\alpha\beta}$ is a contribution to the thermal noise due to the operating pump. For an adiabatic pump the first term is thus independent of frequency whereas the second term is linear in the pump frequency ω [see, Eqs. (21), (22)].

In Eq. (20b) we have introduced the time averaged conductances

$$\overline{\left|S_{\alpha\beta}(E)\right|^2} = \int\limits_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left|S_{\alpha\beta}(E,t)\right|^2.$$

In Eq. (20c) we have introduced the quantities which are intrinsic characteristics of a working pump: the instantaneous currents generated by the pump.[71] The first one dI_α/dE is the spectral density of (dc) currents pushed by the pump into the lead α:

$$\frac{dI_\alpha(E)}{dE} = i\frac{e}{2\pi}\int\limits_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left(\frac{\partial\hat{S}}{\partial t}\frac{\partial\hat{S}^\dagger}{\partial E} - \frac{\partial\hat{S}}{\partial E}\frac{\partial\hat{S}^\dagger}{\partial t}\right)_{\alpha\alpha}. \tag{21}$$

The second quantity is a symmetrized spectral current density describing currents driven from lead α to lead β: $dI^{(s)}_{\alpha\beta}/dE = dI_{\alpha\beta}/dE + dI_{\beta\alpha}/dE$, where $dI_{\alpha\beta}/dE$ is the spectral current density pushed by the pump from lead β to lead α. Without magnetic fields the scattering matrix $\hat{S}$ is symmetric in the lead indices and the matrix $\hat{A}$ is antisymmetric. In this case we find:[71]

$$\frac{dI^{(s)}_{\alpha\beta}(E)}{dE} = i\frac{e}{2\pi}\int\limits_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left(\frac{\partial S_{\alpha\beta}}{\partial t}\frac{\partial S^*_{\alpha\beta}}{\partial E} - \frac{\partial S_{\alpha\beta}}{\partial E}\frac{\partial S^*_{\alpha\beta}}{\partial t}\right). \tag{22}$$

Clearly, we have $\sum_\beta dI^{(s)}_{\alpha\beta}/dE = dI_\alpha/dE$. Therefore each of the terms $\mathbf{P}^{(th,0)}_{\alpha\beta}$ and $\mathbf{P}^{(th,p)}_{\alpha\beta}$ separately satisfies the conservation laws Eq. (11).

The contribution $\mathbf{P}^{(th,p)}_{\alpha\beta}$ is the result of an interference between fluctuating currents coming from the reservoirs and ac currents generated by the pump. Interference terms of this type occur also for a pump in the presence of oscillating reservoir potentials.[34] Such interference terms are thus present whenever the reservoirs are brought into a non-equilibrium state either through ac-voltages or as here through spontaneous equilibrium fluctuations. These interference corrections are small compared to the equilibrium thermal noise,

$$\frac{\mathbf{P}^{(th,p)}_{\alpha\beta}}{\mathbf{P}^{(th,0)}_{\alpha\beta}} \sim \frac{\hbar\omega}{\delta E}. \tag{23}$$

Nevertheless it is these interference corrections which are responsible for the dependence of the high temperature current noise, Eq. (29), on the pump frequency ω.

The pump induced contribution to the thermal noise $\mathbf{P}^{(th,p)}_{\alpha\beta}$ is linear in frequency ω like the pumped current. On the other hand it is linear in temperature T like the equilibrium noise itself. However unlike the $\mathbf{P}^{(th,0)}_{\alpha\beta}$, which is positive at $\alpha=\beta$ and negative at $\alpha\neq\beta$, the interference corrections $\mathbf{P}^{(th,p)}_{\alpha\beta}$ being small have no definite sign. The instantaneous currents $dI_{\alpha\beta}/dE$ produced by the pump can be either positive (flowing from lead β to lead α) or negative (flowing from lead α to lead β). For instance, in Sec.V we consider a particular example where the pump produces a negative contribution to the auto correlations ($\alpha=\beta$) and a positive correction to the cross correlations ($\alpha\neq\beta$). We emphasize that this fact does not change the sign of the whole thermal noise $\mathbf{P}_{\alpha\beta} = \mathbf{P}^{(th,0)}_{\alpha\beta} + \mathbf{P}^{(th,p)}_{\alpha\beta}$ which satisfies the sign rules for fermionic systems.[90] This follows from the general equation (10b).

B. Current shot noise

1. Zero temperature

At zero temperature $T=0$, in the zero-frequency limit considered here, only a working pump is a source of a noise.

The noise arises because the pump generates a non-thermal distribution of outgoing particles with a characteristic energy scale of order $\hbar\omega$. This noise is quite analogous (but not identical) to the shot noise arising in dc-biased (with $eV \sim \hbar\omega$) conductors.[45,90]

Substituting the adiabatic expansion for the Floquet scattering matrix Eq. (16) in Eq. (10) and keeping only leading in ω terms we get the zero temperature noise power as follows:

$$\mathbf{P}_{\alpha\beta} = \frac{2e^2}{h}\sum_{q=1}^{\infty} q\hbar\omega C_{\alpha\beta,q}^{(sym)}(\mu), \tag{24a}$$

$$C_{\alpha\beta,q}^{(sym)}(E) = \frac{C_{\alpha\beta,q}(E) + C_{\alpha\beta,-q}(E)}{2}, \tag{24b}$$

$$C_{\alpha\beta,q}(E) = \sum_{\gamma}\sum_{\delta}\left(S_{\alpha\gamma}^{*}(E)S_{\alpha\delta}(E)\right)_q\left(S_{\beta\delta}^{*}(E)S_{\beta\gamma}(E)\right)_{-q}. \tag{24c}$$

Here we have introduced the matrix $\hat{C}_q$ which we relate to a two-particle scattering matrix in Appendix, Sec. 2. The factor $q\hbar\omega$, with $q = |n-m|$, is the size of an energy window for electrons where the conditions for quantum mechanical exchange (see below) which is responsible for the shot noise are fulfilled.

In Eq. (24a) the matrix elements $C_{\alpha\beta,q}^{(sym)}(\mu)$ are evaluated at the Fermi energy: $E = \mu$. Integrating over energy we take into account the fact that the adiabatic scattering matrix by definition [see, Eq. (15)] is independent of energy over the scale of order $\hbar\omega$. The lower index $\pm q$ denotes the Fourier coefficients.

To clarify the structure of the expression given above, let us consider the scattering process relevant for the zero frequency noise. The basic process consists of scattering of two (non correlated) particles.[90] These particles with energies $E_n = E + n\hbar\omega$ and $E_m = E + m\hbar\omega$ enter the sample through the leads γ and δ. Due to the interaction with the time-dependent scatterer they emit/absorb some energy quanta $\hbar\omega$ and leave the scatterer with energies E and E_p through the leads α and β.

The origin of noise at zero temperature is a quantum mechanical exchange. This effect matters for those outgoing energies E for which simultaneously one incoming state, say corresponding to the energy E_n in lead γ, is full (there is an incident particle) and another incoming state, corresponding to the energy E_m in lead δ, is empty (there is an incident "hole") or vice versa. Note, in general, the incoming states have different energies $E_n \neq E_m$. Therefore these states can be subject to the exchange effect only if the energy difference $q\hbar\omega = |E_n - E_m|$ will be compensated by the oscillating scatterer. Hence we are dealing with *photon-assisted exchange*. As a consequence of quantum mechanical exchange, the particle can appear with energy E in lead α or with energy E_p in lead β.

Note that Eq. (24) is only the leading term of the series expansion of the noise in powers of ω. In principle the approximation Eq. (16) is sufficient to calculate the next term of the zero temperature noise power which is of order $\hbar\omega^2/\delta E$.

2. High temperature: $k_BT \gg \hbar\omega$

At high temperature $k_BT \gg \hbar\omega$ the shot noise is only a part of the entire noise. In this case the term $\mathbf{P}_{\alpha\beta}^{(sh)}$ Eq. (10c) gives (in leading order) rise to a term proportional to $\sim \omega^2$ in the current correlations. Strictly speaking we are unable to calculate the entire noise with an accuracy of order ω^2. To this end one needs to know the Floquet scattering matrix with the same accuracy. This is beyond the approximation Eq. (16) used here. In particular, the term $\mathbf{P}_{\alpha\beta}^{(th)}$, Eq. (10b), would lead to noise of the order $\sim k_BT\left(\hbar\omega/\delta E\right)^2$. However the shot noise Eq. (10c) has a characteristic temperature dependence (it is proportional to the inverse temperature) and thus can be distinguished in experiment from the thermal ($\sim T$) noise.

Calculating Eq. (10c) to leading order in ω we use the lowest adiabatic approximation for the Floquet scattering matrix: $\hat{S}_F(E_n, E_m) \approx \hat{S}_{n-m}(E)$, and find

$$\mathbf{P}_{\alpha\beta}^{(sh)} = \frac{2e^2}{h}\int_0^{\infty} dE\left(\frac{\partial f_0}{\partial E}\right)^2\sum_{q=1}^{\infty}(q\hbar\omega)^2 C_{\alpha\beta,q}^{(sym)}(E). \tag{25}$$

This quadratic in ω noise power is (for $\alpha = \beta$) exactly what was calculated by Avron et al.[14].

3. Weakly energy dependent scattering matrix

If the temperature T is small compared to the relevant energy scale δE for the scattering matrix

$$k_BT \ll \delta E, \tag{26}$$

then integrating over energy in Eq. (10) we can keep the scattering matrix as energy independent. As a result we obtain expression for the shot noise valid at arbitrary ratio $\hbar\omega/k_BT$.

Since the shot noise itself is proportional to ω then to calculate it in leading order in ω we can use the Floquet scattering matrix in the lowest adiabatic approximation. Thus the leading order shot noise is:

$$\mathbf{P}_{\alpha\beta}^{(sh)} = \frac{2e^2}{h}\sum_{q=1}^{\infty} F(q\hbar\omega, k_BT)C_{\alpha\beta,q}^{(sym)}(\mu), \tag{27a}$$

$$F(q\hbar\omega, k_BT) = q\hbar\omega\coth\left(\frac{q\hbar\omega}{2k_BT}\right) - 2k_BT, \tag{27b}$$

The first part on the RHS of Eq. (27b) is well known (with $q = 1$) from the theory of frequency dependent noise[90] and from the fluctuation-dissipation theorem [see, e.g., Ref. 93]. The second part on the RHS of Eq. (27b) (i.e., $-2k_BT$) just compensates the high temperature contribution included already into the thermal noise Eqs. (10b) and (20b).

The equation (27a) reproduces both the equation (24a) at low temperatures $k_BT \ll \hbar\omega$ and the equation (25) at high temperatures $k_BT \gg \hbar\omega$.

The shot noise, Eq. (27), satisfies the conservation laws Eq. (11). That follows directly from the sum rule for the matrix $\hat{C}(\tau)$, Eq. (A.7).

The sign rule for the shot noise follows from Eq. (12). However one can verify it directly in Eq. (27a). To this end we perform the inverse Fourier transformation and represent the shot noise, Eq. (27a), in terms of a time integral:

$$\begin{gathered}\mathbf{P}_{\alpha\beta}^{(sh)} = \tfrac{2e^2}{h}\hbar\omega\int_0^{\mathcal{T}}\tfrac{d\tau}{\mathcal{T}}K(\tau)\Big(\delta_{\alpha\beta} - C_{\alpha\beta}^{(sym)}(\mu,\tau)\Big), \\ K(\tau) = \left(\tfrac{\pi k_BT}{\hbar\omega}\right)^2\sum_{k=-\infty}^{\infty}\sinh^{-2}\left(\tfrac{\pi k_BT}{\hbar\omega}\left[2\pi k + \omega\tau\right]\right).\end{gathered} \tag{28}$$

Since $C_{\alpha\beta}(\tau)$ is positively defined [see, Eq. (A.6)] and it is less then unity [see, Eq. (A.7)], the shot noise $\mathbf{P}^{(sh)}_{\alpha\beta}$ is positive at $\alpha=\beta$ and negative at $\alpha\neq\beta$.

At zero temperature $T=0$, we have $K(\tau)=\frac{1}{2}[1-\cos(\omega\tau)]^{-1}$, and for $\alpha=\beta$ the equation (28) agrees with that obtained within the framework of the theory of full counting statistics.[75,76]

C. Frequency dependence of the pump current noise

At zero temperature the pump produces only shot noise, Eq. (24a), which is linear in pump frequency ω. At nonzero temperatures the pump modifies the thermal noise and produces corrections to it which depend on pump frequency. Therefore the full ω-dependent noise $\delta\mathbf{P}_{\alpha\beta}$ due to the working pump is a sum of two contributions: $\delta\mathbf{P}_{\alpha\beta}=\mathbf{P}^{(th,p)}_{\alpha\beta}+\mathbf{P}^{(sh)}_{\alpha\beta}$. The first one is a correction to the thermal noise, Eq. (20c), (integrated over energy). This contribution is linear in pump frequency. The second one is a high temperature shot noise Eq. (25) which is quadratic in pump frequency. Let us compare them. The additional thermal noise produced by the adiabatic ($\hbar\omega\ll\delta E$) pump is of order $\mathbf{P}^{(th,p)}_{\alpha\beta}\sim k_BT\frac{\hbar\omega}{\delta E}$, where δE is the energy scale of $\hat{S}(E)$. The high temperature shot noise is of the order of $\mathbf{P}^{(sh)}_{\alpha\beta}\sim\frac{(\hbar\omega)^2}{k_BT}$. Their ratio is $\mathbf{P}^{(sh)}_{\alpha\beta}/\mathbf{P}^{(th,p)}_{\alpha\beta}\sim\frac{\hbar\omega\delta E}{(k_BT)^2}$. At relatively high temperatures, $k_BT\geq\delta E$, it is $\mathbf{P}^{(sh)}_{\alpha\beta}/\mathbf{P}^{(th,p)}_{\alpha\beta}\sim\frac{\hbar\omega}{k_BT}$.

Therefore at lower temperatures the shot noise ($\sim\omega^2$) still prevails whereas at higher temperatures the pump produces mainly a linear in ω thermal-like noise.

Thus we expect that with increasing temperature the dependence on the pump frequency of the experimentally detectable additional noise power changes from linear to quadratic and again to linear:

$$\delta\mathbf{P}_{\alpha\beta}\sim\begin{cases}\omega, & k_BT\ll\hbar\omega,\\ \omega^2, & \hbar\omega\ll k_BT\ll\sqrt{\hbar\omega\delta E},\\ \omega, & \sqrt{\hbar\omega\delta E}\ll k_BT.\end{cases}\tag{29}$$

We reemphasize that different physical mechanisms are responsible for the linear in ω behavior of the noise power at zero and at high temperatures. In the former case it is a shot noise whereas in the latter case it is a thermal-like noise.

IV. ADIABATIC HEAT FLOW FLUCTUATIONS: WEAKLY ENERGY DEPENDENT SCATTERING MATRIX

Like in the previous section we use the adiabatic approximation given by Eqs. (16) and (18). We calculate the heat flow noise power Eq. (14) at relatively low temperatures: $k_BT\ll\delta E$. In this case, when integrating over energy in Eq. (14), we can treat the scattering matrix as energy independent and take it at the Fermi energy μ. We calculate the noise to leading order in the pump frequency ω.

A. Heat thermal noise

If the temperature exceeds the pump frequency $k_BT\gg\hbar\omega$, Eq. (19), then the thermal noise $\mathbf{P}^{(th)}_{E,\alpha\beta}$ dominates over the shot noise $\mathbf{P}^{(sh)}_{E,\alpha\beta}$. The heat thermal noise consists of two parts: an equilibrium Nyquist - Johnson heat flow noise[95] and a linear in ω correction due to an operating pump. The last term is similar to a thermal current noise, Eq. (20).

Keeping $\hat{S}(E)\approx$ const we can relate the heat thermal noise $\mathbf{P}^{(th)}_{E,\alpha\beta}$, Eq. (14b), (integrated over energy) to the current thermal noise $\mathbf{P}^{(th)}_{\alpha\beta}$, Eq. (20a) in a very simple way

$$\frac{\mathbf{P}^{(th)}_{E,\alpha\beta}}{\mathbf{P}^{(th)}_{\alpha\beta}}=\frac{\pi^2}{3e^2}(k_BT)^2,\quad k_BT\ll\delta E.\tag{30}$$

Such a ratio is quite expected for the equilibrium thermal noises. That follows from the fluctuation-dissipation theorem (FDT) [see, e.g., Ref. 93] relating the equilibrium fluctuations and the corresponding linear response functions. According to the FDT the current and heat fluctuations in the equilibrium state are proportional to the (linear response) electrical σ and thermal κ conductivity, respectively. If electrons are subject to only elastic scattering then the above conductivities are related through the Wiedemann-Franz law: $\frac{\kappa}{\sigma}=\frac{\pi^2}{3e^2}(k_BT)^2$ [see, e.g., Ref. 94]. For the mesoscopic (phase coherent) samples with energy independent scattering matrix the Wiedemann-Franz law (with the same ratio) can be formulated in terms of electrical G and thermal $\mathcal{K}$ conductances which characterize the entire sample.[96,97,98] As a result we arrive at equation (30).

It should be noted that the equation (30) holds for the thermal noise modified by the pump. Therefore we can suggest that the instantaneous currents dI_α/dE Eq. (21) generated by the pump can be viewed as quasi-equilibrium currents (at least at high temperature).

B. Heat flow shot noise

1. Low temperature: $k_BT\ll\hbar\omega$

When the temperature is lowered below the energy of the modulation quanta $\hbar\omega$ the shot noise $\mathbf{P}^{(sh)}_{E,\alpha\beta}$ becomes dominant. This noise is more subtle and can not be related in a simple way to the current shot noise $\mathbf{P}^{(sh)}_{\alpha\beta}$, Eq. (10c), even for an energy independent scattering matrix. The reason is that unlike the current shot noise the heat flow shot noise, Eq. (14c), is sensitive to the actual energies of outgoing particles.

To clarify this difference we integrate over energy in Eq. (14c). We can do this since the adiabatic scattering matrix is independent of energy over the interval relevant for such an integration at $k_BT\ll\hbar\omega$. Introducing the difference between the energies of incoming states $|E_n-E_m|=q\hbar\omega$ we find:

$$\int\limits_0^\infty dE(E-\mu)(E_p-\mu)\frac{[f_0(E_n)-f_0(E_m)]^2}{2}=$$
$$q\hbar\omega\left(-\frac{(E_n-E_m)^2}{12}+\frac{(E-E_n)(E_p-E_n)+(E-E_m)(E_p-E_m)}{4}\right).\tag{31}$$

The first term (equal to $-(q\hbar\omega)^2/12$) in the big round brackets on the RHS of Eq. (31) depends only on one energy difference. Therefore the corresponding part of the heat flow noise can be represented as a sum over q by analogy with the current shot noise, Eq. (24). In contrast, the second term on the RHS of Eq. (31) depends on two energy differences and thus it results in fluctuations of a completely different nature.

After these preliminary remarks let us proceed with the calculation of the heat flow shot noise, Eq. (14c). To obtain the noise to leading order in the pump frequency ω we use the lowest order adiabatic approximation for the Floquet scattering matrix elements (e.g., $S_{F,\alpha\beta}(E_p, E_n) \approx S_{\alpha\beta,p-n}$, etc.). Taking into account Eq. (31) and applying the inverse Fourier transformation (for all the indices except the last one) we obtain:

$$\mathbf{P}^{(sh)}_{E,\alpha\beta} = \frac{2}{h}\sum_{q=1}^{\infty} q\hbar\omega\left(-\frac{(q\hbar\omega)^2}{6}C^{(sym)}_{\alpha\beta,q}(\mu)+D^{(sym)}_{\alpha\beta,q}(\mu)\right). \quad (32)$$

As in the expression for the current shot noise Eq. (24), the heat current shot noise contains the factor $q\hbar\omega$ which gives the size of the energy window for electrons where the appropriate conditions for quantum mechanical exchange are fulfilled.

Equation (32) for the heat flow shot noise consists of two parts related directly to the two terms of Eq. (31). The first one is proportional to the matrix element $C^{(sym)}_{\alpha\beta,q}$ entering the expression for the current shot noise, Eq. (24a). The second part of Eq. (32) is determined by the new matrix $\hat{D}^{(sym)}_q$. The symmetrized elements are (see also, Appendix, Sec. 3):

$$D^{(sym)}_{\alpha\beta,q} = \frac{D_{\alpha\beta,q}+D_{\alpha\beta,-q}}{2},$$

$$D_{\alpha\beta,q} = \frac{\hbar^2}{2}\sum_{\gamma}\sum_{\delta}\left\{\left(S^*_{\alpha\gamma}\frac{\partial S_{\alpha\delta}}{\partial t}\right)_q\left(S^*_{\beta\gamma}\frac{\partial S_{\beta\delta}}{\partial t}\right)^*_q + \left(\frac{\partial S^*_{\alpha\gamma}}{\partial t}S_{\alpha\delta}\right)_q\left(\frac{\partial S^*_{\beta\gamma}}{\partial t}S_{\beta\delta}\right)^*_q\right\}. \quad (33)$$

The matrix $\hat{D}^{(sym)}_q$ differs essentially from the matrix $\hat{C}^{(sym)}_q$. If the latter is defined by the two-particle scattering matrix alone, Eq. (A.6), the former, in addition, depends on the energy shift matrix, Eqs. (A.15) - (A.17). This allows us to consider the second term in Eq. (32) as a manifestation of a novel effect which is not visible in the current shot noise, Eq. (24a).

Although both parts in Eq. (32) originate from quantum mechanical exchange the energy constraints are accounted for differently in the two terms. In the first part (proportional to $\hat{C}^{(sym)}_q$) quantum mechanical averaging is decoupled from averaging over energy (for the case when the scattering matrix can be taken to be energy independent). Therefore we can interpret it in the same way as the current noise, Eq. (24). In contrast, the second part (proportional to $\hat{D}^{(sym)}_q$) depends on the energy of both the incoming and out-going particles. These energy can be different owing to the pump compensating such difference. In this part averaging over energy affects essentially the result of quantum mechanical averaging.

Concluding this section we briefly discuss the dependence of the current-to-noise ratio on the pump frequency ω. The pumped current and the zero temperature current noise both are linear in the pump frequency. Therefore their ratio is independent of ω. In contrast, the dc heat flow and its fluctuations depend on ω in different ways. While the zero temperature dc heat flow $I_{E,\alpha}$ is quadratic in pump frequency [see, e.g.,Refs. 22,45], the zero temperature heat flow noise is proportional to ω^3, Eq. (32). Therefore the ratio $\mathbf{P}_{E,\alpha\alpha}/I_{E,\alpha} \sim \omega$ can be made arbitrary small in the adiabatic limit $\omega \to 0$.

2. High temperature: $k_BT \gg \hbar\omega$

With increasing temperature the difference between the current and heat flow shot noises diminishes.

If the approximation of an energy independent scattering matrix, Eq. (26), is appropriate, then the high temperature heat flow shot noise is proportional to the current shot noise. Integrating over energy in Eqs. (14c) and Eq. (25) we get

$$\frac{\mathbf{P}^{(sh)}_{E,\alpha\beta}}{\mathbf{P}^{(sh)}_{\alpha\beta}} = \frac{3a}{e^2}(k_BT)^2, \quad \hbar\omega \ll k_BT \ll \delta E. \quad (34)$$

Here the factor is: $a = \int_{-\infty}^{\infty} dx x^2\cosh^{-4}(x) \approx 0.43$. Note, the ratio in Eq. (34) differs from the ratio in Eq. (30) only by the factor of $9a/(\pi)^2 \approx 0.39$.

V. A SIMPLE EXAMPLE

In this section we illustrate the results obtained in sections III and IV performing numerical calculations for a simple model.

Characterization of the noise generated by pumping is of interest for all types of pumps but is of special interest in connection with quantized charge pumping. Nearly quantized charge pumping can be realized near resonant transmission peaks.[37,38,41,43] Therefore it is instructive to investigate a model which exhibits a resonance-like transmission.

We consider a one-dimensional scatterer consisting of two delta function barriers $V_j(x.t),\ j=1,2$ oscillating with frequency ω and located at $x=-L/2$ and $x=L/2$:[99]

$$\begin{aligned} V_1(x,t) &= (V_{01}+2V_{11}\cos(\omega t+\varphi_1))\delta(x+\tfrac{L}{2}),\\ V_2(x,t) &= (V_{02}+2V_{12}\cos(\omega t+\varphi_2))\delta(x-\tfrac{L}{2}), \end{aligned} \quad (35)$$

Thus V_{01} and V_{02} determine the static strength of the barriers whereas V_{11} and V_{12} are the oscillatory amplitudes which are used to pump charge.

To calculate the noise power in the adiabatic limit it is enough to know the stationary scattering matrix $\hat{S}$. Since the adiabatic condition $\hbar\omega \ll \delta E$ should hold over the whole pump cycle, which in our case crosses the resonance line, as a relevant energy scale δE we choose the smallest one characteristic for the scattering matrix, i.e., the width of a transmission resonance.

For the model under consideration the stationary scattering matrix is the following:

$$\hat{S} = \frac{e^{ikL}}{\Delta}\begin{pmatrix} \xi + 2\frac{p_2}{k}\sin(kL) & 1 \\ 1 & \xi + 2\frac{p_1}{k}\sin(kL)\end{pmatrix}. \quad (36)$$

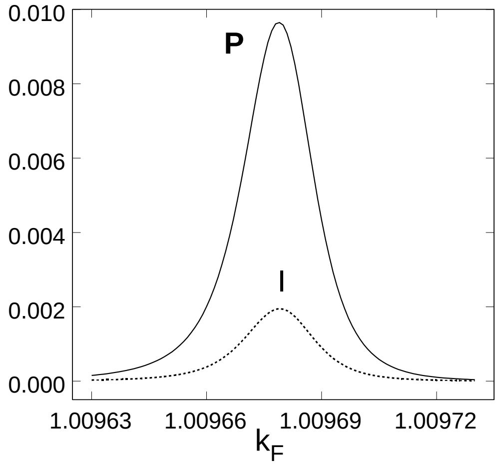

FIG. 2: Current noise. Zero temperature weak pumping. The dependence of the dc current $I \equiv I_1$, in units of $e\omega/(2\pi)$, and the noise power $\mathbf{P} \equiv \mathbf{P}_{11}$, in units of $e^2\omega/\pi$, on the Fermi wave number k_F. The transmission resonance through the double barrier structure is at $k_F = 1.00968$. The parameters are: $L = 100\pi$; $V_{01} = V_{02} = 20$; $V_{11} = V_{12} = 0.1$; φ_2 - φ_1 = $\pi/2$.

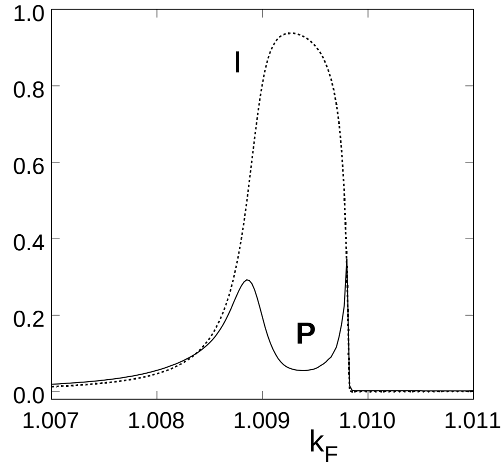

FIG. 3: Current noise. Zero temperature strong pumping. Parameters are the same as in Fig.2 except that $V_{11} = V_{12} = 10$. Note the different scales in comparison to Fig. 2 .

Here $k = \sqrt{\frac{2m}{\hbar^2}E}$; $p_j = V_j m/\hbar^2$ (j = 1,2); $\xi = (1-\Delta)e^{-ikL}$; $\Delta = 1 + \frac{p_1 p_2}{k^2}(e^{2ikL} - 1) + i\frac{p_1+p_2}{k}$. In numerical calculations we use the units $2m = \hbar = e = 1$. We take $L = 100\pi$; $V_{01} = V_{02} = 20$ and investigate pumping for Fermi energies far exceeding an interlevel distance. We chose it close to the 101st resonance state of the well which is at $k_F = 1.00968$.

A. Current noise power

In the two channels case ($\alpha, \beta = 1, 2$) the current noise power matrix $\hat{\mathbf{P}}$ can be entirely characterized by only one matrix element.[90] This is because of the conservation law Eq. (11). To be definite we consider current auto correlations $\mathbf{P}_{11}$.

1. *Zero temperature*

At zero temperature only the shot noise matters. In Fig.2 and Fig.3 we present the noise power $\mathbf{P} \equiv \mathbf{P}_{11}$ together with the dc pumped current $I \equiv I_1$ as a function of the Fermi wave number $k_F = \sqrt{\frac{2m}{\hbar^2}\mu}$ in the vicinity of a transmission resonance for weak ($V_{1j} \ll V_{0j}$) and strong ($V_{1j} \sim V_{0j}$) pumping regimes, respectively. Note that at zero temperature the pumped current I_α is:[22]

$$I_\alpha = \omega \frac{e}{2\pi} \sum_\beta \sum_{q=-\infty}^{\infty} q \left|S_{\alpha\beta,q}(\mu)\right|^2 . \tag{37}$$

We see that with increasing amplitude V_{1j} of the oscillatory pump parameters the magnitude of the noise power increases. At the same time relative to the pumped current the noise power decreases. In addition, in the strong pumping regime the noise shows a non monotonic behavior in the vicinity of the transmission resonance. Note also that the scale of the vertical axis is very different in Fig.2 and Fig.3. While for a small amplitude pump cycle we explore only the scattering properties in the immediate vicinity of the resonance, the large amplitude pump cycle can enclose the resonance even if the center of the pump cycle is already "far" from resonance. For large amplitude pumping the maximum pumped current is achieved if the resonance lies well in the interior of the pump cycle.

To characterize the noise efficiency of the pump we use the Fano factor F [see, e.g., Ref. 90] which is the ratio of the actual

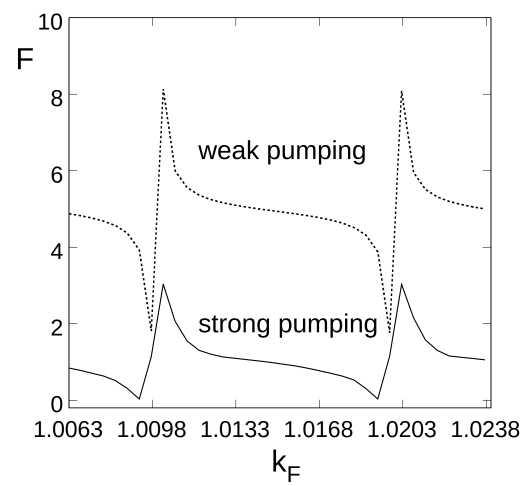

FIG. 4: Current noise. The Fano factor for weak (dashed line) and strong (solid line) pumping at zero temperature. Parameters are the same as in Figs.2 and 3. The transmission resonances through the double barrier structure are at $k_F = 1.00968$ and at $k_F = 1.019677$.

noise power $\mathbf{P}_{\alpha\alpha}$ and the Poisson noise value $\mathbf{P}_{\alpha}^{(P)} = |2eI|$ corresponding to the dc pumped current $I \equiv I_1$. We calculate the following

$$F = \frac{\mathbf{P}_{11}}{|2eI_1|} \tag{38}$$

In Fig.4 we present the Fano factor F vs. k_F. Note that qualitatively we find the same dependence for weak (dashed line) and strong (solid line) pumping. However in the strong pumping regime the Fano factor is smaller. Out of resonance the Fano factor is close to unity. Close to resonance where the charge pumped for a cycle $Q = I\mathcal{T} \approx e$ is approximately quantized the Fano factor approaches zero that is fully consistent with existing discussions[14,75,76] in the literature.

Note that the large value of the Fano factor, Eq. (38), in the weak pumping regime has a simple explanation. In fact, the Fano factor, as defined in Eq. (38), does not reflect the underlying physics of the pump effect. The working pump generates non-equilibrium quasi-electron-hole pairs[45] which dissolve into quasi-electrons and quasi-holes after leaving the scattering region. The entire pumped current I consists of two contribution: $I = I^{(e)} - I^{(h)}$. One of them $I^{(e)}$ is due to the flow of quasi-electrons and the other $I^{(h)}$ is due to the flow of quasi-holes. In the weak pumping regime we have $I^{(e)} \approx I^{(h)}$ and $I \ll I^{(e)}, I^{(h)}$. The noise is a sum of statistically independent contributions from electrons $\mathbf{P}^{(e)}$ and holes $\mathbf{P}^{(h)}$ and a term which describes correlations[45] due to the common origin of electron-hole pairs $\mathbf{P}^{(eh)}$. The noise power can be expressed as a sum of these contributions: $\mathbf{P} = \mathbf{P}^{(e)} + \mathbf{P}^{(h)} + \mathbf{P}^{(eh)}$. The Fano factor calculated for each kind of particles, $F^{(x)} = \mathbf{P}^{(x)}/(2eI^{(x)}), x = "e", "h"$ is close to unity. In contrast, the Fano factor, Eq. (38), calculated with respect to the total current I can be much larger, reaching super-Poissonian values.

2. High temperature

At high temperature the pump generates shot noise, Eq. (25), and contributes to the thermal noise, Eq. (20c). These two contributions depend on both the pump frequency ω and the temperature T in different ways. Therefore we consider them separately.

For simplicity we suppose the temperature to be smaller then the width δE of the transmission resonance:

$$\hbar\omega \ll k_B T \ll \delta E. \tag{39}$$

In this case we can safely integrate over energy in Eqs. (20a), (25) and can express the noise power in terms of the scattering matrix elements given at the Fermi energy μ . Performing the inverse Fourier transformation we find the current auto correlations:

$$\mathbf{P}_{\alpha\alpha}^{(sh)} = \frac{e^2\hbar}{6\pi k_B T}\int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \sum_{\delta\neq\alpha} \left| \sum_{\gamma} \frac{\partial S_{\alpha\gamma}}{\partial t} S_{\delta\gamma}^{*} \right|^2 \tag{40a}$$

$$\mathbf{P}_{\alpha\alpha}^{(th,p)} = i\frac{e^2 k_B T}{\pi}\int\limits_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \sum_{\delta\neq\alpha} \left(\frac{\partial S_{\alpha\delta}}{\partial t}\frac{\partial S_{\alpha\delta}^{*}}{\partial E} - \frac{\partial S_{\alpha\delta}}{\partial E}\frac{\partial S_{\alpha\delta}^{*}}{\partial t} \right). \tag{40b}$$

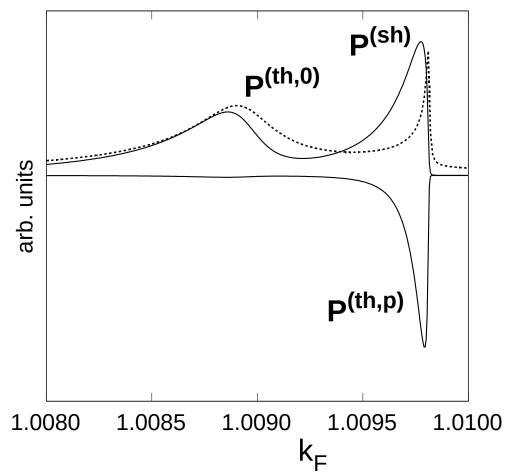

FIG. 5: Current noise. High temperature strong pumping. The dependence of the pump induced contributions to the noise power $\mathbf{P}^{(sh)} \equiv \mathbf{P}_{11}^{(sh)}$, Eq. (40a), and $\mathbf{P}^{(th,p)} \equiv \mathbf{P}_{11}^{(th,p)}$, Eq. (40b), on the Fermi wave number k_F. For comparison the thermal noise $\mathbf{P}^{(th,0)} \equiv \mathbf{P}_{11}^{(th,0)}$, Eq. (20b), (dashed line) is presented as well. All the quantities are given in arbitrary units, Parameters are the same as in Fig.3.

In Fig. 5 we depict $\mathbf{P}^{(th,p)} \equiv \mathbf{P}_{11}^{(th,p)}$ and $\mathbf{P}^{(sh)} \equiv \mathbf{P}_{11}^{(sh)}$ (in arbitrary units) for strong pumping close to the transmission resonance. The thermal noise $\mathbf{P}^{(th,0)} = \frac{4e^2}{h} k_B T \left|S_{12}\right|^2$ is depicted as well.

The behavior of the high temperature shot noise $\mathbf{P}^{(sh)}$ is similar to that of both the zero temperature shot noise, Fig.3, and the thermal noise $\mathbf{P}^{(th,0)}$, Fig.5 (dashed line). In contrast, the pump induced correction to the thermal noise $\mathbf{P}^{(th,p)}$ is essentially different. In the case under consideration, first, it is essentially negative, and, second, it contributes within a narrow energy window.

B. Heat flow noise power at zero temperature

Since the heat flow noise differs from the current noise only at low temperatures $k_B T \ll \hbar\omega$ we present the results of calculations for the heat flow noise only for $T = 0$.

Together with the heat flow noise we calculate the heat current $I_{E,\alpha}$. In the adiabatic case the heat current reads:[22]

$$I_{E,\alpha} = \frac{\hbar\omega^2}{4\pi} \sum_{\beta} \sum_{q=-\infty}^{\infty} q^2 \left|S_{\alpha\beta,q}(\mu)\right|^2. \tag{41}$$

Like the current noise the heat flow noise is essentially different in weak ($V_{1j} \ll V_{0j}$) and strong ($V_{1j} \sim V_{0j}$) pumping regimes.

1. Weak pumping

In this regime the heat flow noise peaks when the Fermi energy μ matches a transmission resonance energy. This is very

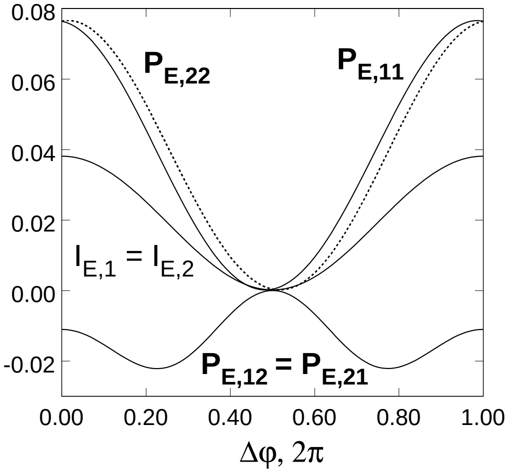

FIG. 6: Heat flow noise at the transmission resonance. Zero temperature weak pumping. The dependence of the dc heat flow $I_{E,1} = I_{E,2}$, in units of $\hbar\omega^2/(4\pi)$, the heat auto correlations $\mathbf{P}_{E,11}$, $\mathbf{P}_{E,22}$, in units of $10\hbar^2\omega^3/\pi$, and the heat cross correlations $\mathbf{P}_{E,12} = \mathbf{P}_{E,21}$, in units of $\hbar^2\omega^3/\pi$, on the phase difference $\Delta\varphi = \varphi_1 - \varphi_2$, in units of 2π. $k_F = 1.00968$. The other parameters are the same as in Fig.2.

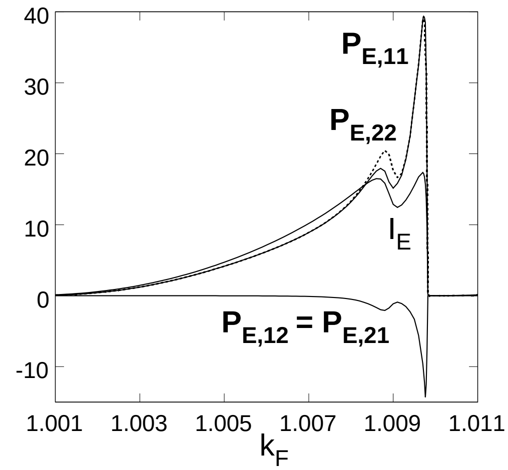

FIG. 7: Heat flow noise. Zero temperature strong pumping. The dependence of the dc heat flows $I_E \equiv I_{E,1} = I_{E,2}$, in units of $\hbar\omega^2/(4\pi)$, and its noise $\mathbf{P}_{E,\alpha\beta}$, in units of $100\hbar^2\omega^3/\pi$, on the Fermi wave number k_F. The transmission resonance through the double barrier structure appears at $k_F = 1.00968$. Parameters are the same as in Fig.3.

similar to the current noise, Fig.2. However the dependence on the phase difference $\varphi_1 - \varphi_2$ clearly shows the difference between the heat flow noise and the current noise. This is related to the fact that the pump is a source of heat flows and thus neither the heat flows nor its fluctuations are subject to the conservation laws, Eq. (11).

Note that the heat flows are sensitive to the spatial asymmetry of the scatterer: if $V_{01} \neq V_{02}$ and/or $V_{11} \neq V_{12}$ then $I_{E,1} \neq I_{E,2}$. Similarly to the heat flow current, the heat flow noise is sensitive to the spatial asymmetry of the scatterer. In addition the heat flow auto correlations are sensitive to a weak spatial asymmetry arising if the time reversal invariance is broken by the working pump.[22] Let us consider a stationary scatterer which is spatially symmetric: $V_{01} = V_{02}$. Further suppose that the potentials V_{01} and V_{02} oscillate with small amplitudes and with a phase lag $\Delta\varphi \equiv \varphi_1 - \varphi_2$. If $\Delta\varphi \neq 0$ then the time reversal invariance is broken. This in turn breaks the spatial symmetry of a double barrier. Note that neither the heat and charge currents nor the current noise do feel this induced spatial asymmetry.

In Fig.6 we present the heat flow auto correlations $\mathbf{P}_{E,jj}, j = 1,2$ and the heat flow cross correlations $\mathbf{P}_{E,12} = \mathbf{P}_{E,21}$ together with the dc heat flow $I_{E,1} = I_{E,2}$ as a function of the phase difference $\varphi_1 - \varphi_2$ for the weak ($V_{1j} \ll V_{0j}$) pumping regime at the transmission resonance. Note the different units for auto correlations and cross correlations.

We see that the heat flow noise cross correlations are negative like the current cross correlations. However unlike the current noise the heat flow auto correlations are unrelated to the cross correlations. In addition the heat flow auto correlations are different at both leads. This is a consequence of the sensitivity to the heat flow noise to the induced spatial asymmetry discussed above.

2. *Strong pumping*

In Fig.7 we present the heat flow noise together with the dc heat flow as a function of the Fermi wave number k_F in the vicinity of the transmission resonance at strong ($V_{1j} \sim V_{0j}$) pumping. Note the increase of the ratio $\mathbf{P}_{12}/\mathbf{P}_{\alpha\alpha}$ compared with the weak pumping regime. In Fig.6 $\mathbf{P}_{12}$ and $\mathbf{P}_{\alpha\alpha}$ are given on different scales whereas in Fig.7 they are given on the same scale.

Close to the transmission resonance the heat noise and its fluctuations show a non monotonic dependence on the Fermi energy μ (the Fermi wave vector k_F). However unlike the current noise, Fig.3, the heat flow noise remains large even at near quantized charge pumping.

C. Breit-Wigner resonance

We conclude this section with a brief discussion of the sharp drop of the pumped current and noise in the strong pumping regime occurring just after the transmission resonance [see, Fig.3, Fig.5, and Fig.7].

At strong pumping the pumped charge is large if the pumping contour encloses the resonance line corresponding to $E = E_F$.[38] For our model, Eq. (35), and for well separated transmission resonances each time only one resonance contributes to the pumped charge and noise. If the Fermi energy E_F changes then the corresponding resonance line [in the plane of pump parameters (V_1, V_2)] can leave the pumping contour [the trace of the point with coordinates $(V_1(t), V_2(t))$]. This leads to a strong decrease of the pumped current. To find the conditions when the resonance line is enclosed by the pumping cycle we proceed as follows.

If the transmission resonances are well separated then only the one closest to the Fermi energy E_F (say, N-th) contributes to the pumped current. In such a case for our purposes we can approximate the scattering matrix $\hat{S}$ Eq. (36) by the Breit-Wigner scattering matrix $\hat{S} \approx \hat{S}_{BW}$:

$$\hat{S}_{BW} = \frac{-1}{\Delta E + i\frac{\Gamma}{2}} \begin{pmatrix} \left(\Delta E + i\frac{\Gamma_2 - \Gamma_1}{2}\right)e^{i\theta_1} & i\sqrt{\Gamma_1\Gamma_2}e^{i\theta} \\ i\sqrt{\Gamma_1\Gamma_2}e^{i\theta} & \left(\Delta E - i\frac{\Gamma_2-\Gamma_1}{2}\right)e^{i\theta_2} \end{pmatrix}$$

The N-th transmission resonance occurs at $E = E_N \equiv \frac{\hbar^2 k_N^2}{2m}$ with $k_N \approx \frac{\pi N}{L}\left(1 - \frac{1}{2L}\left(\frac{1}{p_1} + \frac{1}{p_2}\right)\right)$. Here we introduced the following notations: $\Delta E = E - E_N$; $\Gamma = \Gamma_1 + \Gamma_2$; $\Gamma_j = \frac{E_N k_N}{L p_j^2}, j = 1,2$; $\theta = \pi N + \frac{1}{2}(\theta_1 + \theta_2)$; $\theta_j = \frac{k_N}{p_j}, j = 1,2$. Other parameters are defined after Eq. (36).

The pumping contour encloses the resonance line (at least, in part) if the resonance condition $E_F = E_N$ can be achieved during the pumping cycle. Using the dependence $k_N(p_1,p_2)$ we rewrite the resonance condition as follows ($p_j = mV_j/\hbar^2$):

$$\frac{1}{V_1(t)} + \frac{1}{V_2(t)} = \frac{mL}{\hbar^2 E_{N0}}\left(E_{N0} - E_F\right),$$

where $E_{N0} = \frac{\hbar^2}{2m}\left(\frac{\pi N}{L}\right)^2$. Since all the quantities are positive one can conclude that the above equation has a solution only if $E_F < E_{N0}$. In contrast, at $E_F > E_{N0}$ the resonance line lies fully away from the pumping cycle and the pumped charge is vanishingly small. Therefore with increasing E_F the pumped current (and noise) decreases as soon as the Fermi energy crosses some resonance state E_{N0} in a nearly isolated ($V_j \to \infty$) well.

VI. CONCLUSION

We have generalized the scattering matrix approach to adiabatic pumping in a system of noninteracting spinless fermions of Ref. 45 to the case of a strong amplitude quantum pump. Consequently, our approach takes into account the energy absorption and emission (by modulation quanta $n\hbar\omega$, $n = 1,2,\dots$) of electrons traversing the oscillating scatterer. This allows a description of the correlation properties (noise) of a pump at high ($k_BT \gg \hbar\omega$) as well as at low ($k_BT \ll \hbar\omega$) temperatures. We found that the noise produced by the pump is described by photon-assisted quantum mechanical exchange amplitudes.

At low temperature the pump is the main source of a noise. The low temperature noise is linear in pump frequency ω and it can be viewed as an analog of the shot noise in dc-biased conductors.[90] At high temperature the pump produces characteristic high temperature corrections to the equilibrium Nyquist-Johnson noise. These corrections are quadratic in the pump frequency at lower temperatures but become linear in ω if the temperature increases, Eq. (29). The last contribution is due to existence of instantaneous currents, Eqs. (21) and (22), generated by the pump.

The pump is a source of heat flows.[45] From the scatterer to the reservoirs the energy is carried by electrons traversing the pump. The quantum statistical nature of electron flow leading to current noise results in heat flow fluctuations as well. At low temperature ($k_BT \ll \hbar\omega$) the pump produces a heat flow shot noise which exceeds the equilibrium Nyquist-Johnson heat flow noise.

The theory presented allows a unified description of correlation properties of multi-terminal quantum pumps serving as a source of both current and energy flows at low as well as at relatively high temperatures. We argue that even the adiabatic quantum pump is at zero temperature a far from equilibrium system but it approaches a quasi-equilibrium state as the temperature increases beyond the scale $\hbar\omega$ determined by the pump frequency ω. We found that at low temperature the correlation properties of energy flows differ essentially from the correlation properties of currents generated by the pump. In contrast at high temperature both correlations are related to each other in accordance with the fluctuation dissipation theorem.

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APPENDIX

1. Linear in frequency thermal noise

In this section we present in detail the calculation of the thermal noise, Eq. (10b), within the adiabatic approximation. First, using the adiabatic approximation, Eq. (16), we evaluate the transmission probabilities from the Floquet scattering matrix element with an accuracy of ω:

$$\left|S_{F,\alpha\gamma}(E_n,E)\right|^2 = \left(1 + \frac{n\hbar\omega}{2}\frac{\partial}{\partial E}\right)\left|S_{\alpha\gamma,n}(E)\right|^2 + 2\hbar\omega Re[S^*_{\alpha\gamma,n}(E)A_{\alpha\gamma,n}(E)] + O(\omega^2).$$

Then performing the inverse Fourier transformation we sum over n:

$$\sum_{n=-\infty}^{\infty}\left|S_{F,\alpha\gamma}(E_n,E)\right|^2 = \int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\Bigg\{\left|S_{\alpha\gamma}(E,t)\right|^2 + \frac{i\hbar}{2}\left(\frac{\partial S_{\alpha\gamma}(E,t)}{\partial t}\frac{\partial S^*_{\alpha\gamma}(E,t)}{\partial E} + \frac{\partial^2 S_{\alpha\gamma}}{\partial E\partial t}S^*_{\alpha\gamma}(E,t)\right) + 2\hbar\omega Re[S^*_{\alpha\gamma}(E,t)A_{\alpha\gamma}(E,t)]\Bigg\} + O(\omega^2).$$

To remove the second order derivative we integrate by parts over t. Further using an equality which can be obtained from Eq. (18) (see also Ref.71)

$$4\hbar\omega\sum_\gamma Re[S^*_{0,\alpha\gamma}A_{\alpha\gamma}] = \mathcal{P}\{\hat{S}_0;\hat{S}_0^\dagger\}_{\alpha\alpha},$$

we sum over γ:

$$\sum_\gamma\sum_{n=-\infty}^{\infty}\left|S_{F,\alpha\gamma}(E_n,E)\right|^2 = 1 + i\hbar\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left(\frac{\partial\hat{S}(E,t)}{\partial t}\frac{\partial\hat{S}^\dagger(E,t)}{\partial E} - \frac{\partial\hat{S}(E,t)}{\partial E}\frac{\partial\hat{S}^\dagger(E,t)}{\partial t}\right)_{\alpha\alpha} + O(\omega^2).$$

The remaining part is as follows

$$-\sum_{n=-\infty}^{\infty}\left(\left|S_{F,\alpha\beta}(E_n,E)\right|^2+\left|S_{F,\beta\alpha}(E_n,E)\right|^2\right)=$$

$$-\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left\{\left|S_{\alpha\beta}(E,t)\right|^2+\left|S_{\beta\alpha}(E,t)\right|^2+\frac{h}{e}\frac{dI^{(s)}_{\alpha\beta}(E,t)}{dE}\right\}+O(\omega^2).$$

Here we have introduced a symmetrized spectral current density $dI^{(s)}_{\alpha\beta}/dE = dI_{\alpha\beta}/dE + dI_{\beta\alpha}/dE$ generated by the pump.[71] The spectral current density is:

$$\frac{dI_{\alpha\beta}}{dE}=\frac{e}{h}\Big(2\hbar\omega Re[S^*_{0,\alpha\beta}A_{\alpha\beta}]+\frac{1}{2}\mathcal{P}\{S_{0,\alpha\beta};S^*_{0,\alpha\beta}\}\Big).$$

Finally collecting together all the terms we obtain Eqs. (20).

2. Two-particle scattering matrix

The scattering matrix $\hat{S}$ collects all the quantum mechanical amplitudes corresponding to the scattering of a single particle by the multi-terminal sample. This quantity is appropriate to describe the observable, like a current, which can be represented as a sum over single-particle states.

The noise, the quantity of interest in the present paper, is different. It is a current-current correlation function and, thus, the noise involves essentially two-particle scattering (even for noninteracting particles).[90] To characterize two-particle scattering it is convenient to introduce *a two-particle scattering matrix* $\hat{\Sigma}$.

In a general non-stationary case the scattering matrix $\hat{S}$ depends on two times (an arrival time, and a departure time), and thus the matrix $\hat{\Sigma}$ depends on four times (two times for each particle). However within the adiabatic picture used in the present paper the scattering matrix $\hat{S}(t)$ depends on only one time. It is the scattering matrix *frozen* at the time the particle traverses the scatterer. Therefore within the frozen scattering matrix approximation the matrix $\hat{\Sigma}$ depends on two times only. We define this matrix as follows:

$$\hat{\Sigma}(t_1,t_2)=\hat{S}(t_1)\hat{S}^\dagger(t_2). \tag{A.1}$$

This definition is appropriate for describing of an electron hole scattering process relevant for the zero temperature shot noise, Sec.III B 1. The time arguments determine the scattering matrix encountered by the electron at time t_1 and by the hole at time t_2.

The scattering matrix Eq. (A.1) assumes implicitly that electron and hole enter the scatterer through the same lead. Note if an electron and hole are in the same lead then they have no effect on the average current nor on the zero frequency noise of interest here. Therefore we can say that the scattering matrix $\hat{\Sigma}$ describes *effectively* a two particle scattering process with only an outgoing electron and a hole. These outgoing particles can be viewed as composing an electron-hole pair generated by the pump.

In this Appendix we collect some evident but useful properties of a two-particle frozen scattering matrix $\hat{\Sigma}(t_1,t_2)$.

The matrix $\hat{\Sigma}(t_1,t_2)$ is unitary

$$\hat{\Sigma}(t_1,t_2)\hat{\Sigma}^\dagger(t_1,t_2)=\hat{I}, \tag{A.2}$$

and it is a unit matrix at coincident times

$$\hat{\Sigma}(t_1,t_1)=\hat{I}. \tag{A.3}$$

Here $\hat{I}$ is a unit matrix with matrix elements $I_{\alpha\beta}=\delta_{\alpha\beta}$.

At times which are close to each other: $t_1=t+\Delta t$, $t_2=t$, $\Delta t\to 0$, we can write

$$\hat{\Sigma}(t_1,t)=\hat{I}+\left(t_1-t\right)\partial_{t_1}\hat{\Sigma}(t)+\frac{(t_1-t)^2}{2}\partial^2_{t_1}\hat{\Sigma}(t)+\dots. \tag{A.4}$$

Here we have introduced the abbreviations $\partial^n_{t_1}\hat{\Sigma}(t) = \left.\frac{\partial^n\hat{\Sigma}(t_1,t)}{\partial t_1^n}\right|_{t_1=t} \equiv \frac{\partial^n\hat{S}(t)}{\partial t^n}\hat{S}^\dagger(t)$. The diagonal element $\partial_{t_1}\hat{\Sigma}_{\alpha\alpha}(t)=\frac{-2\pi i}{e}I_\alpha(t)$ is proportional to a zero temperature current I_α pushed by the scatterer into the lead α.

For the scatterer with periodically evolving properties the matrix $\hat{\Sigma}$ is periodic in both arguments:

$$\hat{\Sigma}(t_1,t_2)=\hat{\Sigma}(t_1+\mathcal{T},t_2)=\hat{\Sigma}(t_1,t_2+\mathcal{T}). \tag{A.5}$$

The squared matrix element $|\Sigma_{\alpha\beta}(t_1,t_2)|^2$ is the probability for an electron and a hole to leave the scattering region through the lead α at time moment t_1 and the lead β at time moment t_2, respectively. Averaging it over the middle time $t=\frac{t_1+t_2}{2}$ we find the matrix element of the matrix $\hat{C}(\tau)$ (where $\tau=t_1-t_2$) which defines the correlations of currents produced by the pump:

$$C_{\alpha\beta}(\tau)=\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left|\Sigma_{\alpha\beta}\left(t+\frac{\tau}{2},t-\frac{\tau}{2}\right)\right|^2. \tag{A.6}$$

By definition the matrix $\hat{C}(\tau)$ is real and positive. In addition, since the matrix $\hat{\Sigma}$ is unitary we have

$$\sum_\alpha C_{\alpha\beta}(\tau)=\sum_\beta C_{\alpha\beta}(\tau)=1. \tag{A.7}$$

The matrix $\hat{C}_q$ of the corresponding Fourier coefficients (entering the equation (24) for the noise)

$$\hat{C}_q=\int_0^{\mathcal{T}}\frac{d\tau}{\mathcal{T}}e^{iq\omega\tau}\hat{C}(\tau), \tag{A.8}$$

is a self adjoint matrix

$$\hat{C}_q=\hat{C}_q^\dagger. \tag{A.9}$$

This equation can be easily proven if one expresses $C_{\alpha\beta,q}$ in terms of the (Fourier coefficients for the product of two) elements of the single particle scattering matrix $\hat{S}$ [Eqs. (A.1), (A.6) and (A.8)]:

$$C_{\alpha\beta,q}=\sum_\gamma\sum_\delta\left(S^*_{\alpha\gamma}S_{\alpha\delta}\right)_q\left(S^*_{\beta\gamma}S_{\beta\delta}\right)^*_q. \tag{A.10}$$

Here we used: $A^*_q=(A^*)_{-q}$.

Equation (A.10) leads to the relation: $C_{\alpha\beta,q}=C^*_{\beta\alpha,q}$, which is nothing but Eq. (A.9). In addition, since $C_{\alpha\beta}$ is real and therefore $C_{\alpha\beta,q}=C^*_{\alpha\beta,-q}$, we have $C^*_{\alpha\beta,-q}=C^*_{\beta\alpha,q}$. This means that the matrix $\hat{C}(\tau)$ is subject to the following symmetry condition

$$C_{\alpha\beta}(\tau)=C_{\beta\alpha}(-\tau). \tag{A.11}$$

We would like to emphasize that this condition has nothing to do with a true micro reversibility condition. In a general non-stationary case[22] the micro reversibility implies not only a reversal of the time argument and an interchange of lead indices (or more generally, interchanging of incoming and outgoing channels) but also the change of the relative phases of all relevant time-dependent processes. For instance, if two parameters $X_i = X_{0i}\cos(\omega t + \varphi_i)$, $i = 1,2$ of a scatterer change periodically in time with phase lag $\Delta\varphi = \varphi_1 - \varphi_2 \neq 0$ then the micro reversibility means (in an adiabatic case) an invariance under the following interchange: $(\alpha, \beta, t, \Delta\varphi) \to (\beta, \alpha, -t, -\Delta\varphi)$. In contrast, the matrix $\hat{C}(\tau)$ is invariant already under the substitution: $(\alpha, \beta, \tau) \to (\beta, \alpha, -\tau)$ (note that $\tau = t_1 - t_2$ is a time difference which obviously changes a sign when time is reversed).

If time reversal invariance (TRI) is present: $\hat{C}^{(TRI)}(\tau) = \hat{C}^{(TRI)}(-\tau)$, then it follows from Eq. (A.11) that the matrix $\hat{C}^{(TRI)}$ is symmetric in lead indices

$$C_{\alpha\beta}^{(TRI)} = C_{\beta\alpha}^{(TRI)}. \tag{A.12}$$

Note the symmetrized matrix $\hat{C}^{(sym)}(\tau) = \frac{1}{2}\left(\hat{C}(\tau) + \hat{C}(-\tau)\right)$ defining the zero frequency shot noise power, Eq. (28), is obviously symmetric in lead indices as well

$$C_{\alpha\beta}^{(sym)} = C_{\beta\alpha}^{(sym)}. \tag{A.13}$$

Finally note the useful equality

$$\sum_{q=-\infty}^{\infty} \hat{C}_q = \hat{I}. \tag{A.14}$$

The proof is as follows. Using Eq. (A.10) we get

$$\sum_{q=-\infty}^{\infty} C_{\alpha\beta,q} = \sum_{\gamma}\sum_{\delta}\sum_{q=-\infty}^{\infty}\left(S_{\alpha\gamma}^{*}S_{\alpha\delta}\right)_q \left(S_{\beta\gamma}^{*}S_{\beta\delta}\right)_q^{*}$$

$$= \int_0^{\mathcal{T}} \frac{d\tau}{\mathcal{T}} \sum_{\gamma} S_{\alpha\gamma}^{*}(\tau) S_{\beta\gamma}(\tau) \sum_{\delta} S_{\alpha\delta}(\tau) S_{\beta\delta}^{*}(\tau) = \delta_{\alpha\beta}.$$

Here we used the unitarity condition for the single particle scattering matrix.

3. The matrix $\hat{D}$

We define the matrix $\hat{D}(\tau)$, whose Fourier coefficients enter the expression for the zero temperature heat noise power, Eq. (32), in analogy with the matrix $\hat{C}(\tau)$, Eq. (A.6):

$$D_{\alpha\beta}(t_1 - t_2) = \frac{1}{4}\int_0^{\mathcal{T}} \frac{d(t_1+t_2)}{\mathcal{T}}\left(\Sigma_{\alpha\beta}(t_1,t_2)\Xi_{\alpha\beta}^{*}(t_1,t_2) + c.c.\right). \tag{A.15}$$

Here *c.c.* means complex conjugate terms. Here we have introduced a new matrix:

$$\hat{\Xi}(t_1,t_2) = \hbar^2 \frac{\partial \hat{S}(t_1)}{\partial t_1}\frac{\partial \hat{S}^{\dagger}(t_2)}{\partial t_2}. \tag{A.16}$$

This matrix can be represented as a product of two energy shift matrixes and a two-particle scattering matrix:

$$\hat{\Xi}(t_1,t_2) = \hat{\mathcal{E}}(t_1)\hat{\Sigma}(t_1,t_2)\hat{\mathcal{E}}^{\dagger}(t_2). \tag{A.17}$$

The energy shift matrix $\hat{\mathcal{E}}(t)$ is defined as follows:[14,28]

$$\hat{\mathcal{E}}(t) = i\hbar\frac{\partial \hat{S}(t)}{\partial t}\hat{S}^{\dagger}(t).$$

The matrix $\hat{\Xi}(t_1,t_2)$ emphasizes an importance of energies of particles for the heat flow fluctuations.

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