Nonlinear Dyn
DOI 10.1007/s11071-007-9198-1

ORIGINAL ARTICLE

Transient in a two-DOF nonlinear system

A. Kozmin • Yu. Mikhlin • C. Pierre

Received: 26 April 2006 / Accepted: 13 November 2006

Abstract Dynamics of a system containing a linear oscillator, linearly coupled to an essentially nonlinear attachment, is considered. A damping is taken into account. It is assumed that some initial excitation implies vibrations of the linear oscillator. Envelops of the subsystem's kinetic energies are selected to use the numerical investigation of transient in the system. The parametrical optimization approach is used to obtain regions of effective energy transfer in the system parameter space. It is demonstrated that this efficient energy transfer may be obtained for a rather small value of the attachment mass.

Keywords Transient. Nonlinear energy transfer • Parametric optimization

1 Introduction

The absorption problem is very important in engineering. The classical passive absorber in the form of the single-DOF linear oscillator was used first by Frahm [1] to reduce forced oscillations. He discovered that an extinguishing of the resonance vibration is possible if the fundamental frequency of the absorber is equal to the frequency of the external periodical force. To avoid the resonance vibrations near the fundamental frequencies of the 2-DOF system containing coupled oscillators, Den Hartog [2] used the linear damped absorber. In many cases the absorption can be made effective by using linear absorbers with big masses, but this is impossible to realize in most concrete systems, particularly in civil engineering.

Roberson and Arnold [3, 4] considered the non-damp ed single-DOF absorber connected with a main x system by using the spring with a cubic char-ante is tic. They showed that the soft connecting spring alloi ows to extend the external frequencies interval where thef forced vibration of the main system can be reduced.

Over the past years numerous new devices have been used for the vibration absorption and for the reduction of the transient responses of structures. It seems interesting to study nonlinear passive absorbers for this reduction in nonlinear structures. In literature, nonlinear absorbers have been already used like in [3, 4], or [5] where a pendulum-type centrifugal vibration absorber is used to reduce torsion oscillations, or in [6] where the pendulum is also used as a vibration absorber to reduce response of a flexible cantilever beam, etc. Impact absorber was also considered [7, 8].

A description of the energy transfer was first presented in work by Witt and Gorelik [9]. Authors of this work tried to explain in the point of view of the classical mechanics the effect of the splitting in the spectrum of combination scattering. They considered the spring

A. Kozmin • Yu. Mikhlin (H)
Kharkov Polytechnical Institute, National Technical University, Kharkov, Ukraine
e-mail: muv@kpi.kharkov.ua or yurLmikhlin@mail.ru

C. Pierre
McGill University, Montreal, Canada

pendulum as an example of two nonlinear coupling linear partial subsystems. It was shown that for any small connection the absolute transient of energy of angular oscillations to energy of vertical oscillations and back takes place for the frequency ratio which is equal to $2:1$. Quickness and intensity of this transient is dependent on initial conditions. For some initial conditions, this transient is absent (this is a case of periodical solutions), and the partial subsystems are synchronized.

Transfer of energy in a case of bending and torsional vibrations of elastic systems is described by Kononenko [10]. Transfer of energy was investigated too by Struble and Heinbockel [11, 12], Cheshankov [13], Celman [14], and Mercer et al. [15]. In a book by Starzhinsky [16], the energy transient in Lyapunov systems, and systems close to ones, was analyzed. Dynamics of the spring pendulum, pendulum on the elastic suspension, and elementary particles in cyclic accelerators were considered. It was shown that the problem of the energy transfer groups together with a problem of dynamics of oscillation chain. First stage of analysis is to determine a periodic solution and regions of its instability in the system parameter space by using the general mathematical theory of parametric resonance. Second stage is a determination of bifurcation periodic solutions which appear for some critical parameter values. Asymptotic methods permit to analyze the energy transfer.

The energy transient from one nonlinear vibration mode to another, is considered in a book by Nayfeh and Mook [17]. In [18], it was shown that for some conditions this transfer from high-frequency modes to low-frequency mode is possible in quasilinear systems. But, this transfer is caused by the modes interaction, and the one-way channeling of energy is not obligatory. Transfer of energy from high-frequency to low-frequency modes of the elastic system nonlinear vibrations was described by Hedrih [19, 20].

In papers by Vakakis, Manevitch, Gendelman, Bergman, and others [21-25], theoretical investigation and some experimental verification on the use of nonlinear localization for reducing the transmitted vibrations in structures subjected to transient base motions has been presented. Conditions for the effective energy transfer and localization are discussed. The experimental assembly, containing the main linear subsystem and the nonlinear absorber is described in [24]. Authors of the works [21-25] define the controlling process of the space energy transfer from the place of their initial appearance in the system under consideration to the other specific place where the energy, as the final result, is localized, as the "energy pumping." It means a controled, unidirectional process of the energy transfer to passive nonlinear receiver where the energy is localized and diminishes in time due to damping dissipation. Energetic criterion of this pumping is not proposed. Regions of this energy pumping in parameter space of mechanical systems were also not obtained.

In the presented paper, the authors use effective methods of global optimization to obtain such regions. This paper is organized as follows: in Section 2, the recent methods of global optimization are presented, and the ^-transformation method, used here, is described in detail. Then, in Section 3, a character of transient for two mechanical models, namely, for the spring pendulum and for a system of two linearly connected oscillators, one of them linear, and other nonlinear, is considered. In particular, envelops of the subsystem's kinetic energies are analyzed. Then, some criterion of the full energy transfer from the initially perturbed linear subsystem to the nonlinear attachment, is formulated. By using this criterion, the curves of the full energy transfer in place of the second model parameters, are obtained. This full energy transfer is illustrated by numerical simulation. Additional criterion of optimization, implied with the energy, which returns to the linear subsystem, is discussed and used to obtain corresponding regions in the system parameter place. The character of the energy transfer in 2-DOF system, containing the essentially nonlinear oscillator with a small mass, which corresponds to concrete engineering practice, is studied in Section 4. Regions of the effective energy transfer in the parameter place are obtained. In this section, the influence of degree of nonlinearity and initial energy on the energy transfer is discussed too.

2 Methods of global optimization: Global minimum search by using the method of ^-transformation

There are a lot of works on determination of optimal solutions of theoretical and applied problems, beginning from classic works by Bernoulli, Euler, Lagrange, Hamilton, and others. Considering concrete problems, it is difficult, or, may be, impossible to obtain properties of the input quantities and output parameters analytical dependence. So, there exists a problem of construction methods which permit to determine optimal solutions in a case when assumptions about a nature of function under consideration are absent.

2.1 Existent approaches to solve the global optimization problems

There are no universally effective algorithms to solve the global optimization problems.

All known global optimization methods (GOMs) can be divided into two categories: deterministic and stochastic [26, 27]. Deterministic methods permit to obtain the global solution by using the exhaustive findings over all the problem feasible sets. So, deterministic methods lose an effectiveness if the dimension of the problem increases. Moreover, such methods need in addition a limitation on the criterion function. Stochastic algorithms permit to get away from such problems. A majority of stochastic methods permit to estimate a value of the criterion function in randomized points of the feasible set with the next data processing.

The first method of global optimization was the *Monte-Carlo method.* On the basis of the Monte-Carlo method, the *multistart method* [28] was created. In this method, the same subset consisting of the N points is chosen (stochastically or deterministically) from some set of the problem feasible points. The local descent algorithm is started from each point, and the best solution is chosen from all obtained local solutions. This method is not very effective, because one and the same local solution can be found sometimes. But some me fective GOMs were based of the principal idea of the multistart method .

It is impossible to present here in detail all existing GOMs whose description can be found in different publications. One mentions here some GOMs [26-33], namely, the *clustering method* which is a modification of the multistart method (here a careful selection of points of the local finding is made); the *topographical method*; the *bisection (covering)* and *interval (branch-and-bound) methods;* the *branch-and-bound strategy* which divides the region of finding to a set of many-dimensional cubes; the *tunneling method* (a principal deficiency of the method is a necessity to solve difficult nonlinear differential equations); the principal idea of the method of *simulated annealing* comes from the physics of the liquid freezing; the main idea of the *evolutionary algorithms* is taken from the biological evolution processes; the *controlled random search algorithm* which is a method of the direct search; *trajectory methods,* in particular, the *continuation method.*

2.2 The ^-transformation method

At present, it is impossible to predict which GOM is the most effective. In this paper, one of the most universal and effective GOMs is used, namely, the -transformation method [34].

The main peculiarity of this method is that an object of the analysis is not the criterion function $F(x_1, x_2 x_n)$, but some function ^(Z), which is a result of the function $F(x_1, x_2 x_n)$ transformation. This transformation is based on the definition of partition utilized for the Lebesgue integral construc tion. This conception can be successfully used to solve problems the analysis of which was difficult up to now, in particular, problems with the criterion function $F(x_1, x_2, \ldots, x_n)$ which is not differentiable in a point of extremum.

Let E be some measurable set from the space R^N, and $F(x)$ some function from the class $L_p(E)$. This set can be divided to a sequence of the linked subsets e_{0i} $(i = 1, 2, \ldots, v)$, and on each subset e_{0i} the function $F(x)$ is convex. For the three-dimensional space this situation is illustrated by Fig. 1.

If $E^* = \{x : x \in E,\ F(x) > Z\}$. One defines then as $m(E^*)$ the measure of the set E^*. One introduces the following function:

$$) = m(E^*). \qquad (2.1)$$

This function can be determined by the next Lebesgue integral:

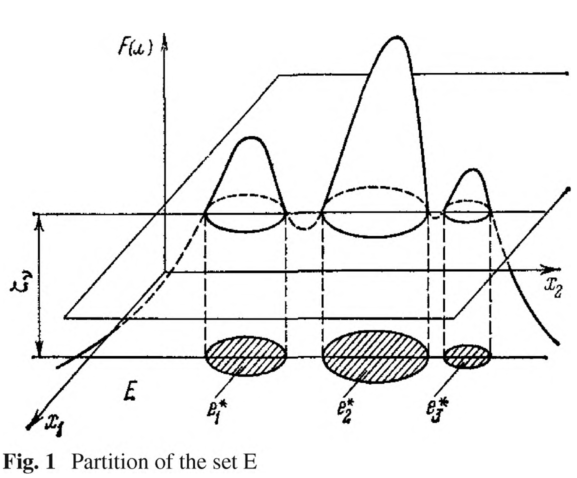


Fig. 1 Partition of the set E

$$m(El) = (L) \quad 1,1 \quad 0_V(x_i, x_n)\,dx\backslash\ldots dx_n \tag{2.2}$$

where $0z(x_i . \ , x_n)$ is the characteristic function of the form

$$6_V(\text{xi}, \quad \begin{matrix} 1, x \ e \ E \\ 0, E \ \backslash \ E'i \end{matrix} \quad Z \tag{2.3}$$

One performs now the Lebesgue's partition of the function $F(x)$. An interval of the $F(x)$ range is divided for a set of subintervals,

$$0 = Z_0 < Z1 < \blacksquare\blacksquare\blacksquare < Zv < \blacksquare\blacksquare\blacksquare < Z^* = \max F(x). \tag{2.4}$$

Note that a finding of the minimum is principally the same.

It can be shown [34] that the transformed function ^(Z) has the following principal properties: the function, independently of the prototype $F(x)$ dimension, is a function of the single variable; the function is monotone and nonincreasing one for all $F(x)$ e $L_p(E)$; a value Z^*, when ^(Z) = 0, is equal to a value of the function $F(x)$ e L_p (E) global extremum, m $\{x : F(x) = \text{Fmax}\} = 0$.

Analytical dependence between the functions $F(x)$ and ^(Z) can be obtained in explicit form only in simplest cases. Other possibility of the function ^(Z) construction consists in its determination in different points, next approximation and extrapolation of the function, and in a determination of the function ^(Z) zero, which corresponds to a value of the function $F(x)$ global extremum.

To determine the function ^(Z) in some point Z_v, statistical tests can be used. It is conditionally assumed that a measure of the set E, on which the function $F(x)$ is defined, is equal to the unit, that is, m $(E) = 1$. Let us define later a value of the measure m $\{x : F(x) > Z_v\}$ with respect to $m(E)$. So, it depends on the determination of some probability, which can be presented in the form of the integral (2.2). The probability P_v can be calculated by the next simple way. In fact, after some number s of statistical tests, it is possible to calculate the number of events $\%_v$, for which $F(x) > Z_v$, and to calculate a frequency or, a statistical probability

$$P_V — H_v/S. \tag{2.5}$$

According to the law of big numbers, if the number s increases, the frequency tends (on probability) to the desired value $P_v = m(E^*)$.

So, the determination of the desired measure of the set E^* consists in conducting of statistical tests for which the variables $x_{1;}$ $x_2, \ldots, xi$ $, \ldots, x_n$ take equiprobable random values. Each test consists in determination of the function F $(x_{1;}$ $x_2, ---, x_n)$ value for given values of independent variables, and in comparison of the value with the given level Z_v. Depending on results of this comparison one has these points x, for which in the v-th test the inequality $F(x) > Z_v$ is satisfied. Then, the approximate value of the measure of the set E^* is determined.

As a result, by using the statistical tests method, one obtains the random function ^(Z) instead of the function ^(Z). Construction of this function permits determination of a scalar value of the global extremum, but coordinates of it remain unknown. But, is it possible to determine all coordinates of the function F $(x_{1;}$ $x_2, \ldots, x_n)$ global extremum from the data obtained in statistical tests, which are used to construct the function ^(Z). For this, the function x(Z) must be determined, where x_t is a mean value of such $xij(i = 1, \ldots, n; \ j = l, \ \ldots, s)$, for which F $(x_{1;}$ $x_2, \ldots x_n) > Z$. Calculation of the mean values $x^\wedge$ for each level of the ith coordinate can be made by the formula A3 (see Appendix). As stated earlier, by substitution a value ζ^*, which is the function $F(x)$ global extremum, to $\bar{x}_i(\zeta)$, one obtains the ith coordinate of the global extremum.

So, to solve the complex many-dimensional optimization problems by using the ^-transformation method, it is necessary to determine values of the functions ^(Z) and $x\backslash$ (Z) in different points Z_v, then, approximating the obtained data, to solve the formulated problem by method of extrapolation.

Algorithm of the ^-transformation method is described in the Appendix.

3 Energy transfer in some mechanical models

3.1 Energy transfer in a spring pendulum

One considers the transfer of energy in a spring pendulum (Fig. 2). This problem was investigated in [9, 16].

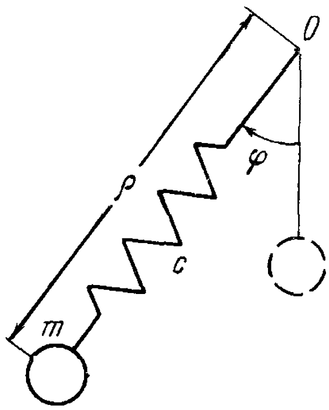


Fig. 2 The spring pendulum

Differential equations of the system motion are the following:

$$p\ \mathbf{0} + \mathbf{2}p\ p\ \mathbf{0} = -g\ p \sin \mathbf{0}$$

$$p - p\,0^2 = \frac{\quad}{m}(\mathrm{p} - l) + g\ \cos \mathbf{0}. \qquad (3.1)$$

It was shown in [16] that for some values of the system parameters a disruption of the vertical vibrations takes place as a result of arbitrary small transversal perturbations.

One develops the numerical investigation of the Equations (3.1) in the point of the energy transfer and in some vicinity of the point. In this system the point is determined by the parameter $y = mg$, and the energy transfer takes place if $y = 0.3333$. Results of thi numerical calculations are presented in Fig. 2. It is pre sented as a dependence of the kinetic energies T_p and $T\mathbf{0}$ of time. Here $T_p = \ _1\ m\ \mathrm{p}^2$ is a kinetic energy of the vertical vibration mode, and $T\$ = \frac{1}{2} m l^2 \mathrm{d}^2$ is akinetic energy of the angular vibration mode. The kinetic energy maximum points are joined. We call corresponding lines as the kinetic energy envelops. In Fig. 3, an envelop of the vertical vibration mode kinetic energy is depicted by solid line, and an envelop of the angular vibration mode kinetic energy is depicted by dot-and-dash line.

We can see from Fig. 3 that in some vicinity of the critical value of the parameter $y = 0.333$, there is a decrease in the kinetic energy T_p amplitude values, and the appearance of a kinetic energy $T^\wedge$ peak. It is clear that all four diagrams illustrate a process of the energy transfer from one vibration mode to the

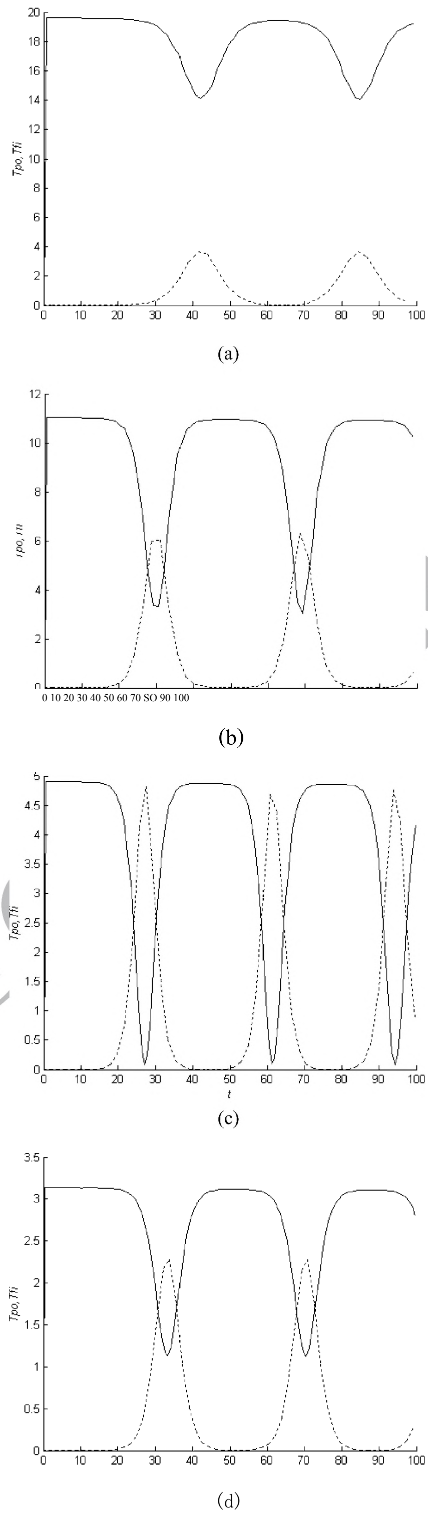


Fig. 3 Diagrams of envelops of kinetic energies of the vertical and angular vibration modes, T_p and $T\$$, for different values of the parameter y: (a) $y = 0.667$, (b) $y = 0.5$, (c) $y = 0.333$, (d) $Y = 0.266$. Here - T_p,--------- Td

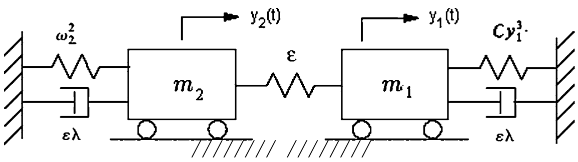


Fig. 4 System of weakly coupled linear and nonlinear oscillators

other. In addition, the full transfer of energy happens for $Y = 0.3333$.

3.2 Energy transfer in 2-DOF system containing an essentially nonlinear oscillator

One considers the energy transfer in a 2-DOF system (Fig. 4), containing the weakly coupled linear and nonlinear oscillators.

One has the following differential equations of motion:

$$m_1 y_i + ek\ y_i + cy_3 + e\ (y_i - y_2) = 0$$

$$m_2 y_2 + ek\ y_2 + C_2\ y_2 + S\ (y_2 - y_i) = 0. \qquad (3.2)$$

Here £ ^ 1, other parameters have an order O (1). Such models were considered in some works, for example, in works by Vakakis, Manevitch et al. Analysis of systems with such essentially nonlinear absorbers permits to select principal advantages of the nonlinear absorption, in particular, the effect of the energy localization. In engineering practice, there is a possibility to obtain nonlinear absorbers with very small linear component in elastic characteristic.

A behavior of the system by using the envelops kinetic energies (these envelops were introduced in Section 3.1) of two subsystems is considered. Later, the adduced values of the kinetic energies of two partial oscillators are presented:

$$T_1 = mT = m\ ^2 2h \quad \frac{\dot{y}_2^2}{}. \qquad (3.3)$$

The kinetic energy envelops obtained for the system (3.2) are shown in Fig. 5 for $m_1 = m_2 = 1$, $X = 0.5$, $c_2 = 0.9$, $c = 5.0$, $£ = 0.1$ and for the next initial values: $y_1(0) = y_2(0) = 0$, $y_1(0) = 0$; $y_2(0) = V_2 h$, where h is the system energy at the moment; $t = 0$ (here $h = 0.8$). Here and later, a dotted line represents an envelop of the adduced kinetic energy of initially perturbed linear oscillator. Solid line represents

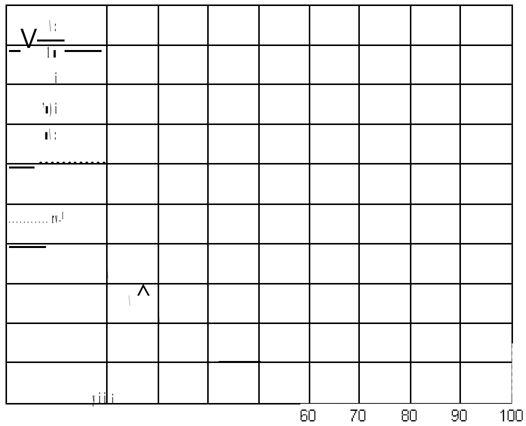


Fig. 5 Diagram of the kinetic energy envelops for the system (3.2)

the same for the nonlinear absorber. Dot-a nd-dash line represents the same for the linear oscil lator without the attached nonlinear subsystem.

By considering this diagram the next problem can be formulated: are there some parameters of the system when the kinetic energy envelop of the linear initially perturbed subsystem turns into zero, and the peak of such envelop of the perturbed nonlinear system simultaneously appears. Such case will be assumed as a case of the full energy transfer.

One selects in Fig. 6 principal parameters which characterize the energy transfer. The point T_{2min} corresponds to the first minimum of the kinetic energy envelop for the initially perturbed linear subsystem. The

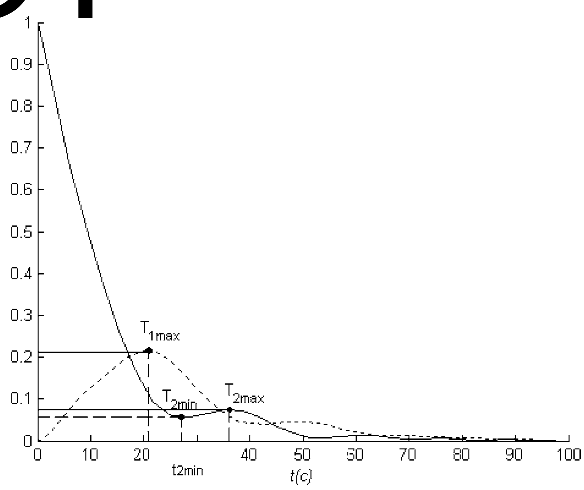


Fig. 6 Principal parameters of a diagram of the kinetic energy envelops

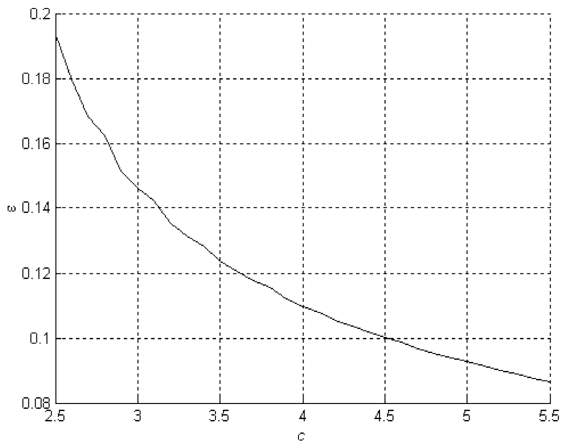


Fig. 7 Curve of the full energy transfer in plane *(s, c)*

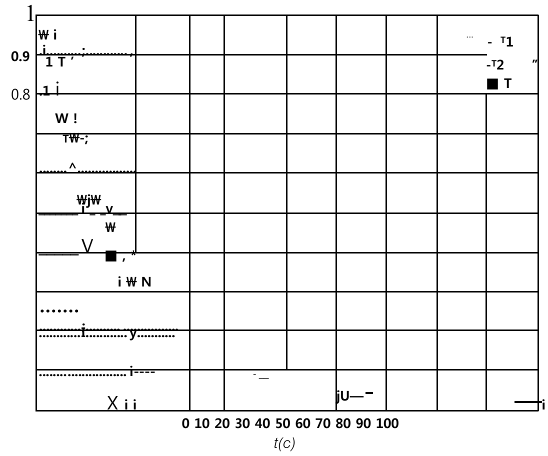


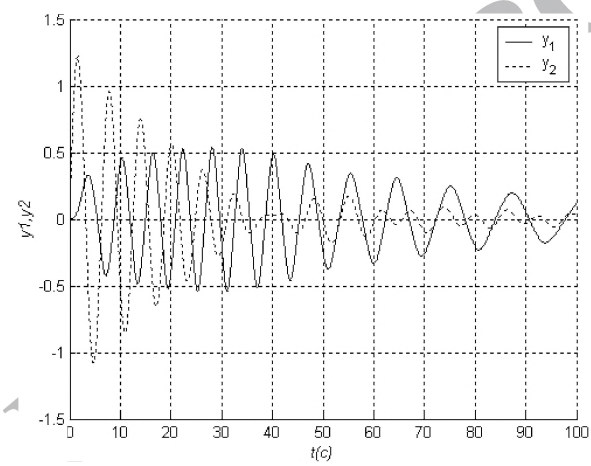
Fig. 8 Envelops of the kinetic energies in point of the full energy transfer (*s* = 0.09268, *c* = 5.0)

point $T_{1\,\max}$ represents a maximum of this envelop of the nonlinear subsystem. The point $T_{2\max}$ represents the second maximum of this envelop for the linear subsystem, and the point $t_{2\min}$ is a time of achievement of the first minimum of this envelop for the perturbed linear subsystem.

One studies in detail the energy transfer in the system (3.2). One takes the system parameters and initial values which were used earlier. By using the proposed criterion of the full energy transfer and the -transformation method, the curve of the full energy transfer in a plane of the system parameters (s, c) was obtained. This curve is shown in Fig. 7.

One selects some point on the curve, and observes a change of the system vibration energy. As previously, the dotted line represents on diagram the envelop of the adduced kinetic energy of the initially perturbed linear oscillator, the solid line represents the same for the nonlinear absorber, and the dot-and-dash line represents the same for the linear oscillator without the attached nonlinear oscillator. Envelops are shown in Fig. 8 for the point of the full energy transfer ($T_{2\min} < 0.0001$), namely, $s = 0.09268$, $c = 5.0$. We can see here that the full energy transfer from the initially perturbed linear oscillator to nonlinear one takes place on the time interval from zero to 40. At the moment $t = 40$ all energy is concentrated in the nonlinear absorber.

The transient in the system (3.2) in point of the full energy transfer is shown on the Fig. 9. We can see that the vibration amplitude of the initially perturbed system decreases up to zero on the time interval from zero to 40. At that time the vibration amplitude of the nonlinear attachment increase appreciably. Beginning with



9 Transient in the system (3.2) in point of the full energy ransfer

some moment close to $t = 30$ the vibration amplitudes of the linear subsystem are smaller than the vibration amplitudes of the attached nonlinear subsystem in three and more times. Then, the back energy transfer to the linear subsystem happens. Vibration amplitudes of the main linear oscillator increase. Then the energy anew leaves the linear subsystem. So, one observes the phenomenon similar to that which was observed in the system (3.1). The sequential transfer from one vibration mode to another one takes place.

Let us see a behavior of the system under consideration, if the parameter s, representing the stiffness of the connected spring, is relatively small. One can see from the Figs. 10 and 11 that in this case the energy transfer from the initially perturbed linear oscillator to

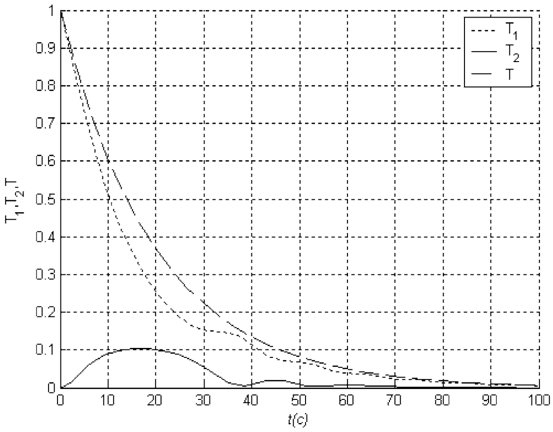


Fig. 10 Kinetic energy envelops for £ = 0.09

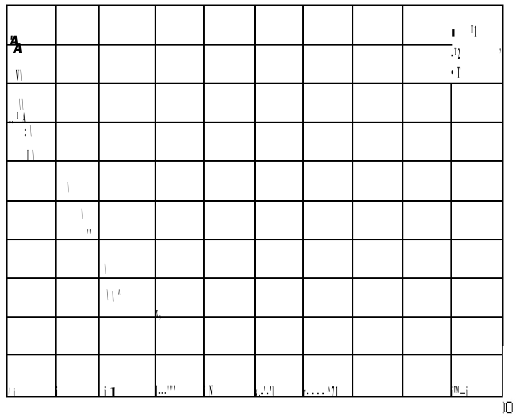


Fig. 12 Kinetic energies envelops for £ = 0.2

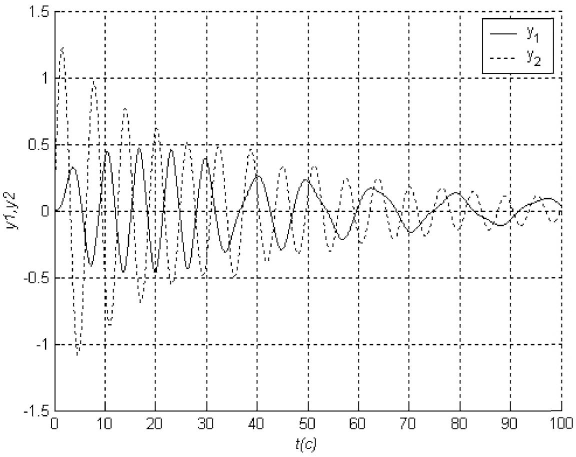


Fig. 11 Transient for e = 0.09

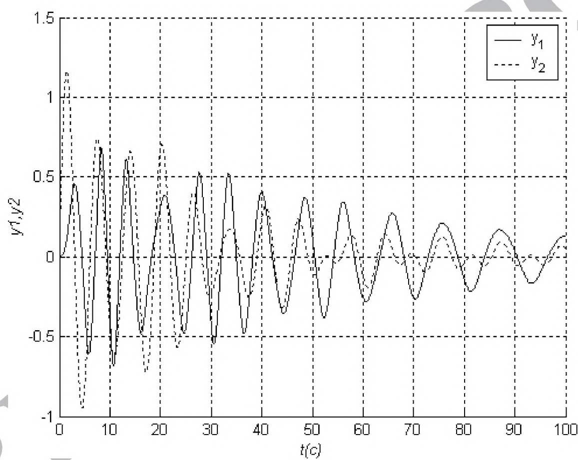


Fig. 13 Transient for $\varepsilon = 0.2$

the nonlinear one is made/becomes weaker. Evidently, this is caused by the weak connection between oscillators, which hinders the energy transfer to the nonlinear attachment.

Let us consider a behavior of the system under consideration, when the parameter £, representing the stiffness of the connected spring, increases (Figs. 12-15).

One has from Figs. 12-15 that an increase of £ leads to more rapid energy transfer from the perturbed oscillator to unperturbed one. A peak of envelop of the unperturbed mass kinetic energy increases together with increasing of £, and it takes place for a smaller time interval. But, in this connection, the full energy transfer from the perturbed oscillator to unperturbed one is absent. There is a significant energy return back to the perturbed oscillator. Figure 14 shows that initially on the time interval from o to10 the rapid transfer of kinetic energy from the perturbed linear oscillator to nonlinear

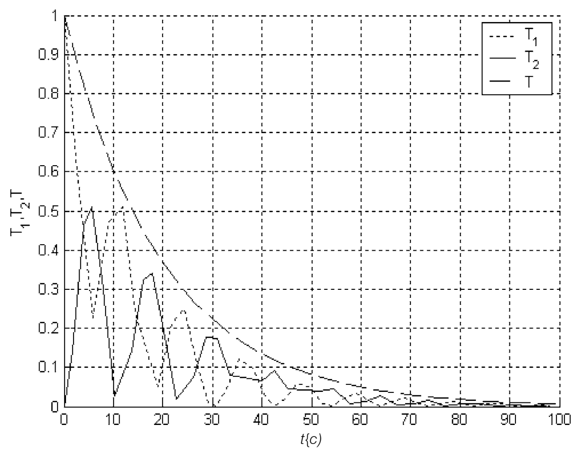


Fig. 14 Kinetic energy envelops for £ = 0.5

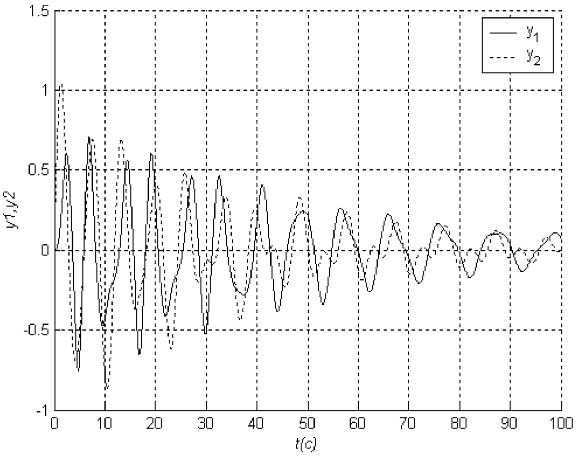


Fig. 15 Transient for $s = 0.5$

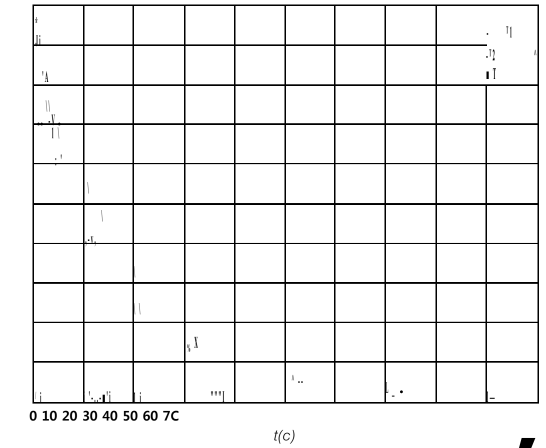
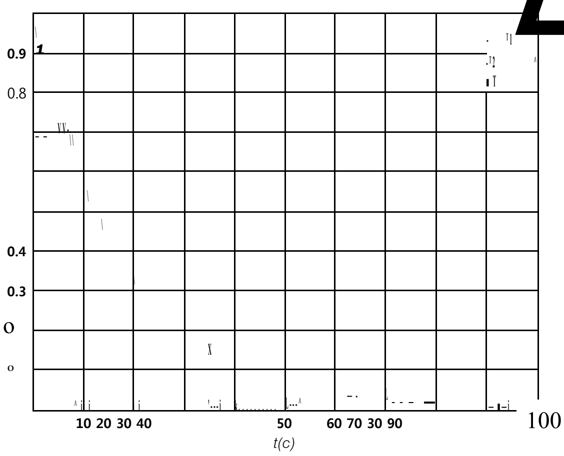


Fig. 16 Envelops of the kinetic energy in point of the full [illegible] transfer, when $c = 2.5$; $s = 0.1933$



17 Envelops of the kinetic energy in point of the full energy ransfer, when $c = 6$; $s = 0.0818$

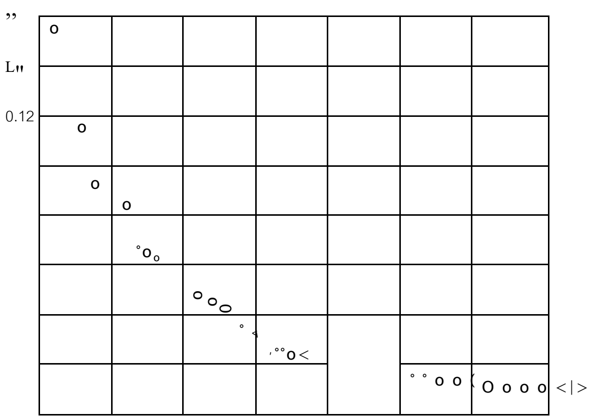


Fig. 18 Relation of T_{2max} depending on c

one takes place. But this is not the full energy transfer. Part of it remains in the perturbed oscillator. Then on the time interval from 10 to 19, this envelop of the kinetic energy of the linear oscillator increases, while the kinetic energy of the nonlinear one sharply decreases.

So, we conclude that an increase of the coefficient s leads to a larger return of the energy. The sequential transfer of the energy from one oscillator to another happens.

One considers anew a curve of the full energy transfer (Fig. 6). Let us analyze the energy envelops when we move to the left or to the right on the curve.

Some part of calculation of envelops of the kinetic energy are shown in Figs. 16 and 17 for different values . of the parameters c and s. It is possible to conclude $_{tran}$* (that a decrease of c and an increase of s leads to a more rapid energy transfer from the initially perturbed linear oscillator to the nonlinear one. But, in this case, the larger return of the energy to the linear subsystem takes place. Increase of c gives us a reduction of the energy return to the linear oscillator.

The conclusion stated earlier is confirmed by Figs. 18 and 19, wherein points of the full energy transfer the relations of the return energy value (after the full energy transfer) $T_{2\ max}$, and a time of the passage time of the full energy transfer t_{2min}, are presented, depending on the change of the parameter c.

One considers now a problem of the energy transfer in the system of two connected oscillators from the point of view of the absorption problem. In this problem the fast energy transfer from the main system to absorber is principal.

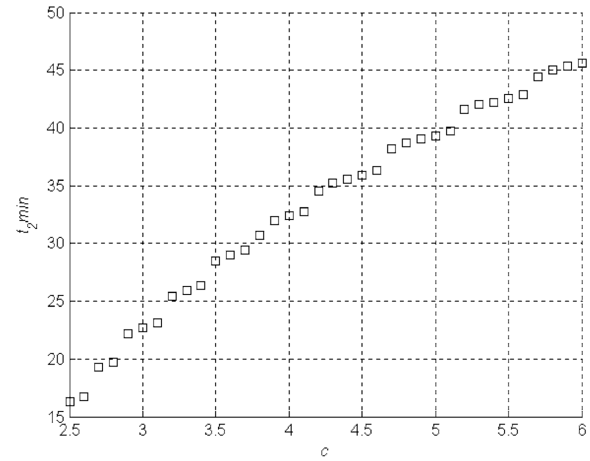


Fig. 19 Relation of $t_{2\min}$ depending on c

Fig. 20 Classification of points of the full energy transfer

Additional limitation to the precedent criterion is introduced: is introduced it needs to minimize too $T_{2\max}$ (Fig. 6), that is the second maximum of the kinetic energy envelop of the initially perturbed linear oscillator. It forms as a result of the energy return from the nonlinear absorber to linear one. Introducing this criterion, we can to classified points of the full energy transfer (Fig. 7) on dependence of $T_{2\max}$. Points on the Fig. 20 represent values of parameters for which $T_{2\max} < 0.5$, triangles represent a case when $0.5 < T_{2\max} < 1$, and circles represent a case when $T_{2\max} > 1$.

Obtained results permit to formulate the next more general problem of the parametric investigation of the energy transfer: to find the values of the parameters c and s, when the envelop of the adduced kinetic energy of the initially perturbed subsystem tends to zero.

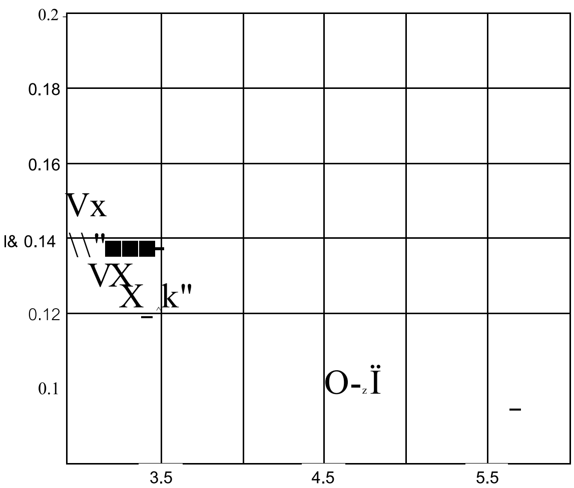


Fig. 21 Region of the parameters c and s, where the energy transfer takes place

In a neighborhood of this minimum the maximum of envelop of the nonlinear absorber exists.

Set of points, obtained by using this additional criterion, are shown in Fig. 21, where the solid line corresponds to a maximal extinguishing of the initially perturbed oscillator energy ($r_{2\min} < 0.0001$). Set of parameters, bounded by dotted lines, corresponds to the limitations $T_{2\min} < 0.05$ and $T_{2\max} < 0.05$. Region of the parameter values where $T_{2mn} < 0.1$ and $T_{2\max} < 0.1$, are limited by dot-and-dash lines.

4 Character of the energy transfer in 2-DOF system containing the essentially nonlinear oscillator with a small mass

One considers now a case, when the attached nonlinear absorber has a mass which is essentially smaller than that of the linear subsystem. It corresponds to the real engineering practice when a use of the absorbers with big mass is impossible. The damping coefficient X is equal here to 0.1.

The energy envelops in a point of the full energy transfer for $m_1 = 0.1$ is shown in Fig. 22, and the corresponding transient is shown in Fig. 23. Note that in Fig. 23, showing the transient, the solid line represents vibrations of the initially perturbed linear oscillator with attached absorber, and the dotted line represents such vibrations without this absorber. We can see here the essential extinguishing of the main linear subsystem

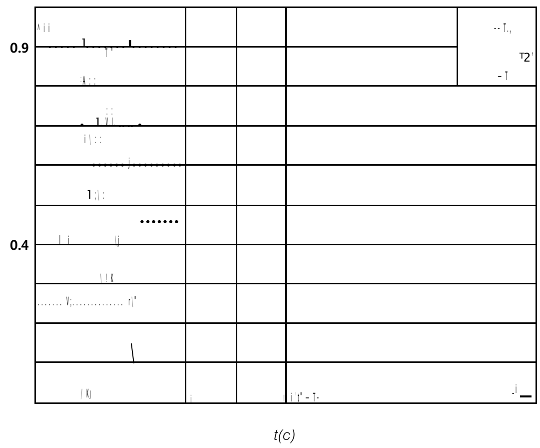


Fig. 22 Energy envelops in a point of the full energy transfer for $m_1 = 0.1$

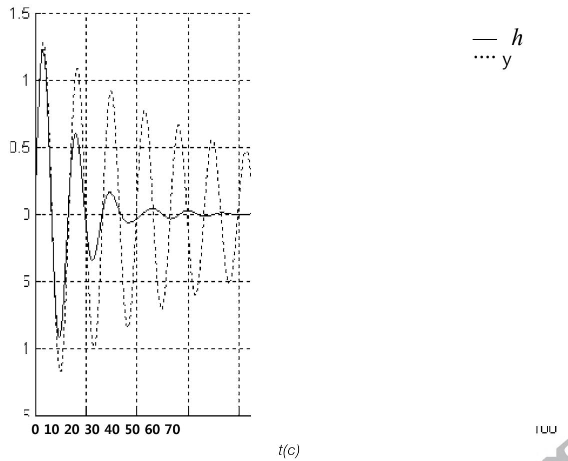


Fig. 23 Transient for linear subsystem with absorber *(solid line)* and without absorber *(dotted line)* when $m_1 = 0.1$

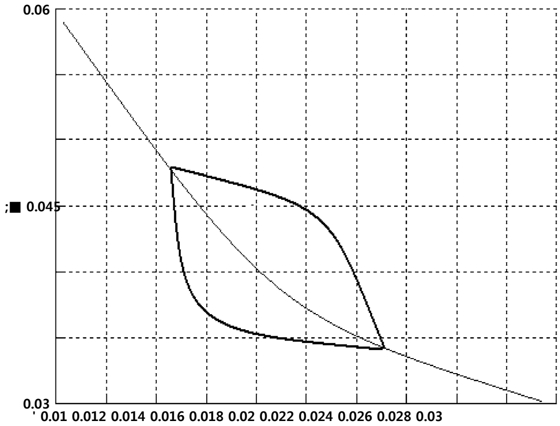


Fig. 24 Region of the effective energy transfer

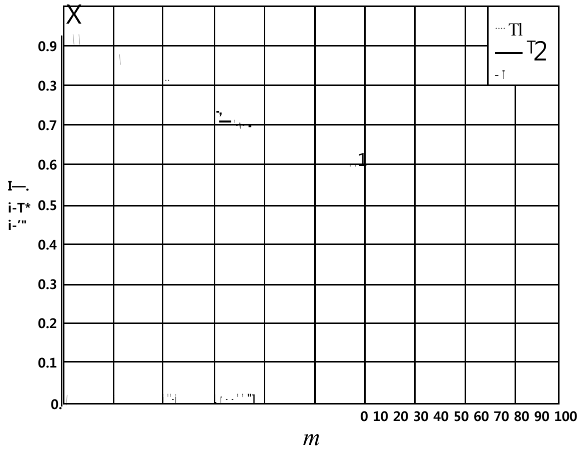


Fig. 25 Envelops of kinetic energies in a point of the full energy tran er (c = 0.018, s = 0.04)

vibrations if this nonlinear absorber exists. So, < ing the mass of absorber, the rapid one-way channeling of energy in the system under consideration is found.

The next criterion of efficiency of the energy transfer (Fig. 24) is now introduced. Namely:

The energy transfer must be effected for a fixed and not large time interval. (For example, here $t = 20$ is introduced). For a quantitative valuation of this transfer one estimates the following: if a loss of energy for the initially perturbed linear oscillator $T^{\wedge}$-Tl during this time interval is equal to 70% or more, the energy transfer is considered as effective (or optimal). Moreover, this condition has to be fulfilled in the point T_{2max}, that is in a point of the second maximum of the kinetic energy T_1 envelop.

For the system (3.2), with the parameters $m_1 = 0.1$, $m_2 = 1$, $X = 0.1$, $c_2 = 0.9$, $s_1 = 0.1$ and initial conditions $j_1(o) = y_2(o) = o$, $,y_1(o) = o$; $y_2(o) =$ V2h, where h = o.8, the region of the effective energy transfer in a place of the parameters (c, s) was obtained. Boundary of this region is marked by boldface line in Fig. 26. Thin line represents a curve of the full energy transfer for this case.

One of the solutions from the region is presented in Figs. 25 and 26.

Investigation of influence of degree of nonlinearity and initial energy to the energy transfer was made too. Namely, three cases for the system (3.2) are considered: (1) the anchor spring is linear; (2) the anchor spring has a cubic nonlinearity; (3) the anchor spring has a fifth degree of nonlinearity. Corresponding points are

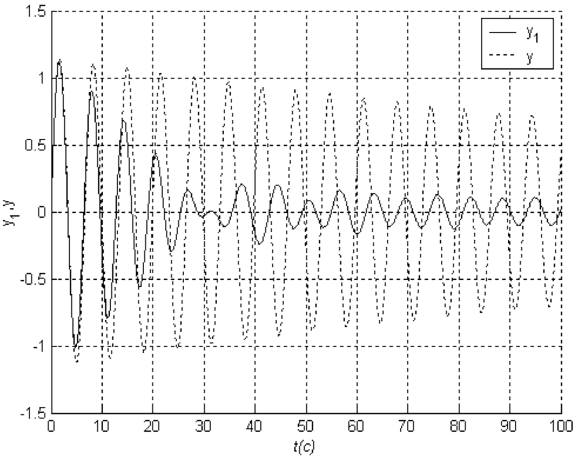


Fig. 26 Transient for $c = 0.018$, $s = 0.04$

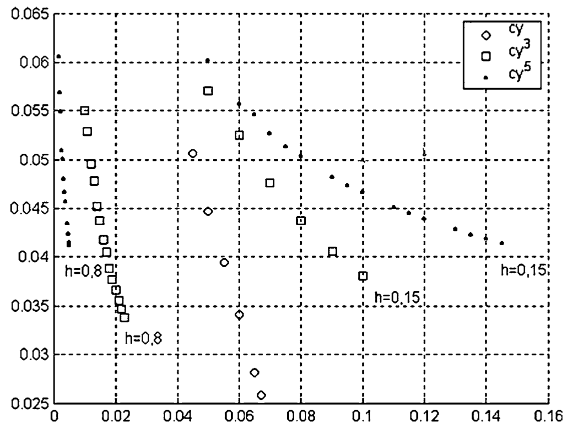


Fig. 27 Points of the full energy transfer in systems with different degree of nonlinearity and different initial energy er-

presented in Fig. 27 by using circles (linear spring), squares (cubic nonlinearity), and black points (fifth-degree nonlinearity) on a place of the parameters (c,s). It is very interesting that points of the full energy transfer for the nonlinear spring are different for different values of the initial energy.

5 Conclusions

In the present study, an analysis of the energy transfer in some 2-DOF nonlinear mechanical systems have been carried out. The effective method of global optimization, namely, the *V*-transformation method, is used here. Principal characteristics of the energy transfer, namely, envelops of the subsystems kinetic energies, are selected to use the numerical investigation of this process. Criterion of the full energy transfer is proposed and discussed. By using this criterion, the curves of the full energy transfer in a place of the system parameters, are obtained. This full energy transfer is illustrated by numerous numerical simulations. Additional criterion of optimization, implied with the energy, which returns to the linear subsystem, is discussed and used to obtain corresponding regions of the effective energy transfer in the system parameter place. Regions of the effective energy transfer in the parameter place are obtained. Influence of degree of nonlinearity and initial energy to the energy transfer is discussed too. It seems that the proposed approach can be used to investigate the transfer of energy in different nonlinear systems.

Appendix

The algorithm of the -transformation] method is presented here.

Input data is the following: a, b are vectors of lower and high limitations of the parameter limitation; $f(...)$ is a procedure of calculation of the minimized function, N is the number of the sketch points.

Output data is the following: x as a point of the minimum, $f(x)$.

1. The values $x_1, x_2, \ldots, xt, \ldots, x_n$ are chosen by random low with uniform distribution.
2. A value of the function $F(x_1, x_2, \ldots, x_n)$ is calculated.
   The points 1 and 2 s times are repeated.
3. The sup F and inf F among s values of the function $F(x_1, x_2, \ldots, x_n)$ are determined.
4. The interval $[(\sup F - \inf F)/2, \sup F]$ is divided into k equal components.
5. For all k, $Zv(v = 1, 2, \ldots, k)$ and ^ are determined according to the formulae:

$$Zv = (\sup F - \inf F)/2 + (v - 1)AZ \quad (A.1)$$

$$Vv = \S v/s, \ AZ = Zv_{-1} - Zv_{-2} \quad (A.2)$$

   where $\S_v$ is a number of events when, for the given $\S_v$, one has $F(x) > Z_v$.
6. The obtained data are approximated so as to determine the parabolic approximation.

7. The formulated problem is solved by calculating the roots Zi and Z_2 of the parabola.
8. The smaller values of the roots are found, corresponding to a scalar value Z^* of the function $F(x_1, x_2, \ldots, x_n)$ global extremum F_{max}.
9. For each Z_v $(v = 1, 2, \ldots, k)$ a mean value Xi $(i = 1, 2, \ldots,$ k) is calculated by the formula:

$$Xi = \quad / Hv \tag{A.3}$$

10. The coefficients of parabolas which approximate the functions $x\backslash\ (Z) =$ (i $= 1, \ldots, k)$ are determined.
11. The value Z^*, obtained in P. 8, is substituted to the expression, which was determined in P. 10, then one determines i th coordinate x^* of the global extremum.
12. The values x^* are substituted to the function F $(x_1, x_2, \ldots, x_n)$ expression and one determines the required value F^*.
13. The value F^*obtainedinP. 12 is compared, with the scalar value of the global extremum Z^*, determined in P. 8.

Equality of both values shows, with some error, that the problem is solved correctly.

References

1. Frahm, H.: Device for Damping Vibrations of Bodies. US Patent 989958, 1909
2. Den Hartog, J.P.: Mechanical Vibrations. McGraw-H] ill. New York (1934)
3. Roberson, R.E.: Synthesis of a nonlinear dynamic vibration absorber. J. Franklin Inst. 254, 205-220 (1952)
4. Arnold, F.R.: Steady-state behavior of systems provided with nonlinear dynamic vibration absorbers. Trans. ASME. J. Appl. Mech. 22, 487-492 (1955)
5. Shaw, S.W., Wiggins, S.: Chaotic motions of a torsional vibration absorber. Trans. ASME. J. Appl. Mech. 55, 952-958 (1988)
6. Cuvalci, O., Ertas, A.: Pendulum as vibration absorber for flexible structures: experiments and theory. Trans. ASME J. Vib. Acoust. 118, 558-566 (1996)
7. Karyeaclis, M.P., Caughey, T.K.: Stability of a semi-active impact damper: Part I—Local behavior. Trans. ASME J. Appl. Mech. 56, 926-929 (1989)
8. Karyeaclis, M.P., Caughey, T.K.: Stability of a semi-active impact damper: Part II—Global behavior. Trans. ASME. J. Appl. Mech. 56, 930-940 (1989)
9. Gorelik, G.S., Witt, A.A.: Swing of an elastic pendulum as an example of two parametrically bound linear vibration systems. J. Tech. Phys. (USSR) 3(2-3), 294-307 (1933) (in Russian)
10. Kononenko, V.O.: On coupled bending-torsional vibrations. Bending-torsional vibrations and critical velocities. Not. USSR Acad. Sci. 2, 194-237 (1953) (in Russian)
11. Struble, R.A., Heinbockel, J.H.: Energy transfer in a beam-pendulum system. Trans. ASME. J. Appl. Mech. 29, 590-592 (1962)
12. Struble, R.A., Heinbockel, J.H.: Resonant oscillations of a beam-pendulum system. Trans. ASME. J. Appl. Mech. 30, 181-188 (1963)
13. Cheshankov, B.I.: Resonance vibrations of a special double pendulum. Appl. Math. Mech. (PMM USSR) 33(6), 1112-1118 (1969)
14. Tsel'man, F.K.: On pumping transfer of energy between non-linearly-coupled oscillators in third-order resonance. Appl. Math. Mech. (PMM USSR) 34(5), 957-962 (1970)
15. Mercer, C.A., Rees, P.L., Fahy, V.J.: Energy flow between two weakly coupled oscillators subject to transient excitation. J. Sound Vib. 15(3), 373-379 (1971)
16. Starzhinskii, V.M.: Applied Methods of Nonlinear Vibrations. Nauka, Moscow (1977) (Russian)
17. Nayfeh, A.H.,Mook, D.: Nonlinear Oscillations. Wiley, New York (1984)
18. Nayfeh, S.A., Nayfeh, A.H.: Energy transfer from high to low-frequency modes in a flexible structure via modulation. ASME J. Vib. Acoust. 116, 203-207 (1994)
19. Hedrih (Stevanovic), K.: Interpretation of the transfer of energy from high-frequency to low-frequency modes by averaging asymptotic method Krilov-Bogolybov-Mitropolsky. In: Proceedings of the Second International Conference on Asymptotics in Mechanics, pp. 97-104. Saint-Petersberg State Marine Technological University, Saint-Petersberg, Russia (1997)
20. Hedrih (Stevanovic), K.: Multifrequency forced vibrations of thin elastic shells. In: CD Proceedings of the Fifth EUROMECH Nonlinear Dynamics Conference, pp. 2417-2426. Eindhoven University of Technology, Eindhoven, The Netherlands (2005)
21. Gendelman,O.V:Transitionofenergytononlinearlocalized mode in highly asymmetric system of nonlinear oscillators. Nonlinear Dyn. 25, 237-253 (2001)
22. Vakakis, A.F., Gendelman, O.: Energy pumping in nonlinear mechanical oscillators: Part I. Dynamics of the underlying Hamiltonian systems. Trans. ASME. J. Appl. Mech. 68, 34-41 (2001)
23. Gendelman, O., Manevitch, L.I., Vakakis, A.F., M'Closkey, R.: Energy pumping in nonlinear mechanical oscillators: Part II. Resonance capture. Trans. ASME. J. Appl. Mech. 68,42-48 (2001)
24. McFarland, D.M., Bergman, L.A., Vakakis, A.F.: Experimental study of non-linear energy pumping occurring at a single fast frequency. Int. J. Non-linear Mech. 40(6), 891-899 (2005)
25. Manevitch, L., Gendelman, O., Musienko, A.I., Vakakis, A.F., Bergman, L.A.: Dynamic interaction of a semi-infinite linear chain of coupled oscillators with a strongly nonlinear end attachment. Physica D 178, 1-18 (2003)
26. Ali, M., Törn, A., Viitanen, S.: Stochastic global optimization: Problem, classes and solution techniques. J. Glob. Optim. 14, 437-447 (1999)

27. Rinnoy Kan, A.H.G., Timmer, G.T.: Stochastic global optimization methods. Math. Program. 39, 27-78 (1987)
28. Törn, A., Viitanen, S.: Topographical global optimization. In: Floudas, C.A., Pardalos, P.M. (eds.) Recent Advances in Global Optimization, pp. 384-398. Princeton University Press, Princeton, NJ (1992)
29. Evtushenko, Yu.G., Potapov, M.A., Korotkich, V.V.: Recent Advances in Global Optimization, pp. 274-297. Princeton University Press, Princeton, NJ (1992)
30. NemhausterG.L., Pruul E.A., RushmeierR.A.: Branch-and-bound and parallel computation: A historical note. Oper. Res. Lett. 7, 65-69 (1988)
31. Gomez, S., Levy, A.V.: The tunneling method applied to global optimization. In: Boggs, P.T. (ed.) Numerical Optimization, pp. 213-244. SIAM, Philadelphia, PA (1985)
32. Michalewicz, Z.: Genetic Algorithms + Data Structures = Evolution Programs, 3rd edn. Springer-Verlag, New York (1996)
33. Diener, I.: Trajectory methods in global optimization. In: Horst, R., Pardalos, P.M. (eds.) Handbook of Global Optimization, pp. 649-668. Kluwer, Dordrecht, The Netherlands (1995)
34. Chichinadze, V.K.: Decision of Non-Convex Problems of Optimization. Nauka, Moscow (1983) (in Russian)

A

Queries to Author

A1: Au: Please note references have been renumbered to provide for sequential arrangement. Please check.
A2: Au: Please check the citation of Fig. 24 for appropriateness