NONLINEAR NORMAL MODES OF VIBRATING MECHANICAL SYSTEMS AND THEIR APPLICATIONS

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Summary. **The p**rincipal concepts of nonlinear normal vibration modes (NNMs) and methods of their analysis are presented. NNMs for forced and parametric vibrations and generalization of the NNMs to continuous systems are considered. Nonlinear localization and transfer of energy are discussed in the light of NNMs. Different engineering applications of NNMs are analyzed.

1. Theoretical basis of nonlinear normal modes

Nonlinear normal modes (NNMs) are periodic motions of specific type, which can be observed in different nonlinear mechanical systems [1-3]. In the normal vibration mode a finite degree-of-freedom system vibrates like a single-degree-of-freedom conservative one. The significance of NNMs for mechanical engineering is determined by the important properties of these motions. In particular forced resonances motions of nonlinear systems occur close to NNMs. Nonlinear localization and transfer of energy can be analyzed using NNMs.

Kauderer [4] was the first who developed quantitative methods for the NNM analysis in two-DOF conservative nonlinear systems. Rosenberg considered ***n***-DOF conservative systems and deduced the first definition of NNMs as "vibrations in unison", i.e., synchronous periodic motions, where all material points of the system reach their maximum and minimum values at the same instant of time [5,6]. He considered wide classes of essentially nonlinear systems, which have nonlinear vibrations modes with straight modal lines. The NNMs based on the determination of modal lines in configuration space, can be called **the Kauderer-Rosenberg nonlinear normal modes**. In general, the NNM modal lines in a configuration space are curvilinear. The power series method to construct the curvilinear trajectories in conservative systems was proposed in [1,7-9]. The non-localized and localized NNMs, bifurcations of the NNMs and global dynamics of the nonlinear systems near NNMs are analyzed in different papers [1-3,10-15]. Pade´ approximations are used to derive the NNMs with arbitrary amplitudes [16].

Shaw and Pierre developed an alternative concept of NNMs for nonlinear dissipative finite-DOF systems [17,18]. Their research was based on the computation of invariant manifolds of motion in phase space. This second type of the NNMs is called **the Shaw-Pierre nonlinear normal modes**.

Generalization of the NNMs concepts to forced, self-exited and parametric vibrations is possible [1,11,19-24]. In particular the Rauscher method and the power-series method for the modal line construction can be used to analyze the NNMs of non-autonomous systems. NNMs in systems with non-smooth characteristics are considered [25-28]. Generalization of the NNMs to continuous systems is made in several publications [29-31].

2. Applications of nonlinear normal modes

NNMs have been used to solve applied problems of mechanical and aerospace engineering [32]. Such vibrations take place in different structures and machines.

The Kauderer- Rosenberg NNMs are applied for the analysis of large amplitude dynamics of finite-DOF nonlinear mechanical systems. In particular, free and forced NNMs are considered in systems with nonlinear absorbers [33-35]. Localized and non-localized NNMs are analyzed in such systems. Applications of the Kauderer-Rosenberg NNMs for discretized systems are also discussed. These systems can be obtained by use of the Galerkin procedure to initial continuous structures. The Kauderer-Rosenberg NNMs are successfully used to analyze large amplitude free and forced vibrations of the cylindrical shells with geometrical nonlinearity [36,37]; cylindrical shells interacting with a fluid [38]; parametric vibrations of cylindrical shells under the action of longitudinal force [39]; shallow arch snap-through motions [40]; vibrations of beams interacting with essential nonlinear absorbers [41].

The Shaw-Pierre NNMs are applied to analyze the dynamics of nonlinear mechanical systems. In particular NNMs are used to analyze dynamics of pre-twisted beams with geometrical nonlinearity [42]; beam parametric vibrations [22]; nonlinear free vibrations of shallow shells with complex base [43]; nonlinear vibrations of the vehicle suspensions [44]. Nonlinear dynamics of an one-disk rotor in two bearings is studied using the Shaw-Pierre NNMs [45-47]. Gyroscopic effects, nonlinear flexible base, inertial forces in supports and internal resonances are taken into account. NNMs of self-oscillations of rotors in short journal bearings are studied [48].

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