NONLINEAR NORMAL MODES AND THEIR INTERACTION IN NON-IDEAL SYSTEMS WITH VIBRATION ABSORBER

Y.V. Mikhlin*, A.A. Klimenko*, K.Y. Plaksiy*
*National Technical University "KPI", Kharkov, Ukraine

Summary. Nonlinear normal vibration modes (NNMs) of the non-ideal systems, where an interaction of source of energy and linear elastic subsystem takes place, are investigated. Systems under consideration contain the nonlinear absorber, which permits to decrease amplitudes of the elastic subsystem vibrations. Interaction of NNMs in vicinity of resonances is analyzed by using the multiple scales method and transformation to a reduced system.

Introduction

The non-ideal systems are characterized by interaction of source of excitation and the elastic sub-system under this excitation. Such systems are named also as systems with limited power supply. The most interesting phenomenon in non-ideal systems is the Sommerfeld effect [1], when the stable resonance regime with large amplitudes of the elastic sub-system can be observed, and a big part of the source energy is leaved to resonance vibrations. Resonance dynamics of the non-ideal system was first analytically described by V.Kononenko [2]. Then investigations of the non-ideal system dynamics were continued and presented in different publications, in particular, in papers [3,4]. A use of vibration absorber permits to reduce large amplitudes of the resonance regime. The non-ideal systems with nonlinear absorber, in particular, with the pendulum absorber, are considered here. The nonlinear normal modes (NNMs) [5-7] and the multiple scale method [3] are used in this investigation. A behavior of the dissipative non-ideal systems having the nonlinear absorber in vicinity of the resonance is described by use of the reduced system with respect to the system energy, an arctangent of the vibration amplitudes ratio, and the phase difference. Interaction of NNMs and an appearance of so-called "transient nonlinear normal modes" are analyzed.

1. Nonlinear normal modes of the non-ideal system

One considers the non-ideal system containing the linear elastic sub-system which oscillates under periodic excitation. Simultaneously this sub-system influences to a source of excitation. Besides, the system under consideration contains the pendulum absorber (Fig.1).

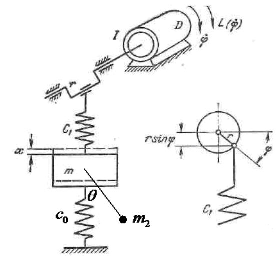

Fig. 1. Non-ideal mechanical system with the pendulum absorber.

Equations of motion of the system are the following:

$$\begin{cases} \left(M+\varepsilon m_2\right)\ddot{x}+\left(c_0+c_1\right)x=c_1 r\sin\varphi-\varepsilon m_2 l\left(\ddot{\theta}\cos\theta-\dot{\theta}^2\sin\theta\right); \\ I\ddot{\varphi}=\varepsilon\left(a-b\dot{\varphi}+c_1 r\left(x-r\sin\varphi\right)\cos\varphi\right); \\ l\ddot{\theta}+g\sin\theta+\ddot{x}\cos\theta=0. \end{cases} \quad (1)$$

In this system two NNMs can be selected: a) a mode of coupled vibrations; b) localized mode when the small vibrations of the elastic sub-system combine with large amplitude vibrations of the absorber. The selected NNMs are determined by construction of trajectories $x=x(\varphi)$, $\theta=\theta(\varphi)$ in the system configuration space. Corresponding equations and boundary conditions at the maximum equipotential surface can be obtained using the NNMs general theory [6,7]. These trajectories are obtained in power series by ε and φ. In particular, trajectory of the mode of coupled vibrations for some values of the system parameters is shown in Fig. 2. A region of instability of the localyzed NNM is narrow, so, this vibration mode is effective for absorption of the elastic system vibrations.

Frequency responses of the resonance vibrations are constructed for a case when the dissipation is taken into account. It is shown that in vicinity of the first resonance the nonlinear absorber permits to reduce vibration amplitudes of the elastic sub-systems in few times. In particular, a frequency response of the elastic sub-system without absorber, obtained for some values of the system parameters, is shown in Fig. 3a; and in the system with the absorber is shown in Fig. 3b.

Influence of vibration of the elastic sub-system to the source of energy is analyzed. It is shown that the system parameters can be chosen so that the system can not be situated in the first resonance region.

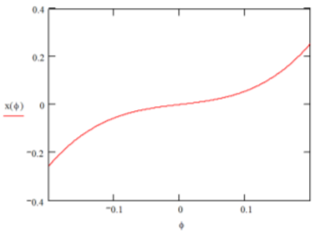

Fig. 2,a. Trajectories on the place of variables x and φ.

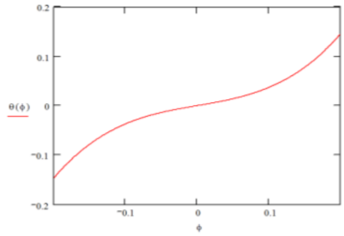

Fig. 2,b. Trajectories on the place of variables θ and φ

Fig. 2. Trajectories of the mode of coupled vibrations.

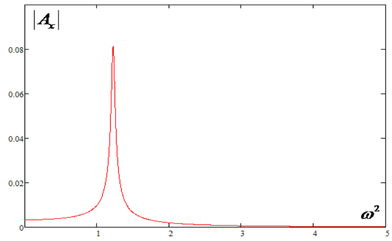

Fig.3a. Frequency response of the system without absorber.

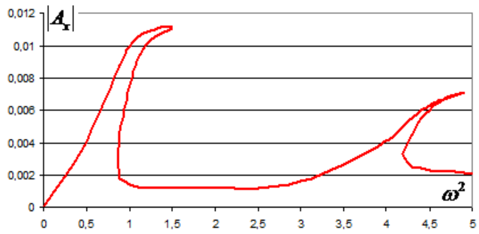

Fig.3б. Frequency response of the system with absorber.

Fig.3. Frequency responses of the elastic sub-system.

2. Dynamics of the non-ideal dissipative system in vicinity of the forced resonances.

Dynamics of the dissipative non-ideal system containing the nonlinear absorber is considered in vicinity of the internal resonances. The multiple scale method and other transforms give the following reduced system for a case of the resonance on the first fundamental frequency:

$$\begin{cases} \dfrac{\partial K}{\partial T_1} = \dfrac{L-A-B\Omega}{2\Omega} K\sin^2\psi + \dfrac{S}{2\omega_2} K\cos^2\psi - \left(\dfrac{C}{2\Omega} K^2 \sin^2\psi + \dfrac{N}{2\Omega}\right)\cos b_1 \sin\psi \\ \dfrac{\partial \psi}{\partial T_1} = \left(\dfrac{L-A-B\Omega}{2\Omega} - \dfrac{S}{2\omega_2}\right)\sin\psi\cos\psi - \left(\dfrac{C}{2\Omega} K\sin^2\psi + \dfrac{N}{2\Omega K}\right)\cos\psi\cos b_1 \\ \dfrac{\partial b_1}{\partial T_1} = -\dfrac{\Delta + \Omega T_0(A+B\Omega)}{2\Omega} - \dfrac{T_0 C}{2} K\sin\psi\cos b_1 + \dfrac{M}{2\Omega}K^2\sin^2\psi + \dfrac{P}{2\Omega}K^2\cos^2\psi + \dfrac{N}{2\Omega K\sin\psi}\sin b_1 \\ \dfrac{\partial b_2}{\partial T_1} = -\dfrac{R}{2\omega_2}K^2\cos^2\psi - \dfrac{T}{2\omega_2}K^2\sin^2\psi, \end{cases} \quad (2)$$

where K is the system energy, decreasing in time; ψ is an arctangent of the vibration amplitudes ratio; an equation with respect to the phase difference, $\varphi = b_1 - b_2$, can be wrote as

$$\varphi' = -\frac{\Delta + T_0(A+B\Omega)}{2\Omega} - \frac{T_0 C}{2\Omega} K\sin\psi\cos b_1 + \frac{M}{2\Omega}K^2\sin^2\psi + \frac{P}{2\Omega}K^2\cos^2\psi + \frac{N}{2\Omega K\sin\psi}\sin b_1 + \frac{R}{2\omega_2}K^2\cos^2\psi + \frac{T}{2\omega_2}K^2\sin^2\psi \quad (3)$$

Dependence of the angle velocity of the rotor Ω for the non-stationary regime is the following: $\frac{\partial\Omega}{\partial T_1} = \frac{1}{2}(A + B\Omega + Ca_1\cos b_1)$. All coefficients in last equations depend of the system parameters. Interaction of NNMs is studied. The *transient nonlinear normal modes*, which are realized only for some determined levels of the dissipative system energy, are observed. In vicinity of time, corresponding to this energy value, the system motions are close to the transient vibration modes; then, when the energy decreases, the system motions tend to other stable vibration mode. Conditions of the energy localization in the nonlinear vibration absorber are obtained.

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