# NONLINEAR NORMAL MODES IN NON-IDEAL SYSTEMS AND ABSORPTION OF THE RESONANCE VIBRATIONS

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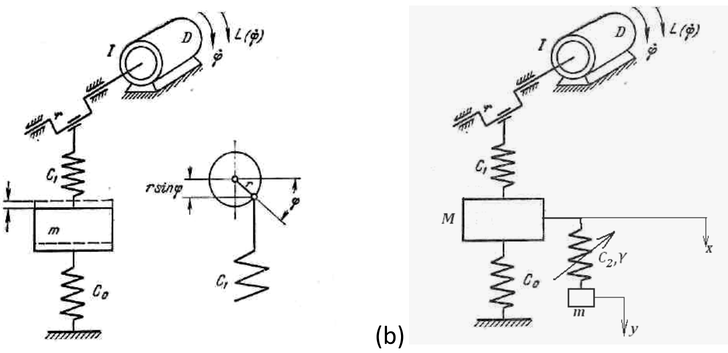
**The system under consideration.** One considers the system which describes an interaction of some rotating subsystem and the linear elastic one. A model of this system is presented in Fig. 1a. The same system with the nonlinear asorber is presented in Fig. 1.b. Equations of motion of the system without absorber are the following:

$$\begin{cases} m\ddot{x} + \beta\dot{x} + cx = c_1 r\sin(\varphi) \\ I\ddot{\varphi} = L(\dot{\varphi}) - H(\dot{\varphi}) + c_1 r(x - r\sin(\varphi))\cos(\varphi) \end{cases} \tag{1}$$

Here the function $L(\dot{\varphi})$ is a controlled torque of the unbalanced rotor of DC motor; $H(\dot{\varphi})$ is a resistance torque of the rotor. The system under consideration is known as non-ideal system [1-3]. It means that the excitation is influenced by the response of the supporting elastic substructure and that the energy source has a limited power supply (non-ideal excitation). The feedback in this system can not be negligible.




(a) (b)

**Figure 1.** (a) The nonlinear system with limited power supply. (b) The non-ideal system with the nonlinear absorber

The equations (1) may be simplified when we accept the torques as linear. One has in this case: $L(\dot{\varphi}) - H(\dot{\varphi}) = A - B\dot{\varphi}$. Introducing the new dimensionless variables, $y = x/r, \tau = \omega t$, and the next parameters

$$M = A/(I\omega^2), N = B/(I\omega^2), \varepsilon q = c_1 r^2/(I\omega^2),\ \varepsilon K = c_1/(m\omega^2), \varepsilon h = \beta/(m\omega)$$

(here $\varepsilon$ is a formal small parameter) one can rewrite the equations (1) as

$$\begin{aligned} & y'' + \varepsilon h y' + \omega^2 y = \varepsilon K \sin\varphi \\ & I\varphi'' = M - N\omega + \varepsilon q(y\cos\varphi - 0.5\sin 2\varphi) \end{aligned} \qquad (2)$$

Here prime denotes a derivation by $\tau$.

**Stationary regimes of the non-ideal system.** Introducing the new "amplitude-phase" variables as

$$y = A\cos(\varphi+\Psi)\ ;\quad y' = -A\sin(\varphi+\Psi)\ ;\quad \varphi' = \Theta,$$

one has the following system:

$$\begin{cases} A' = \dfrac{\varepsilon}{\Theta}\left[K\sin\varphi + hA\sin(\varphi+\Psi)\right]\sin(\varphi+\Psi) \\ \Psi' = -\dfrac{(\Theta-1)}{\Theta} - \dfrac{\varepsilon}{A\Theta}\left[K\sin\varphi + hA\sin(\varphi+\Psi)\right]\cos(\varphi+\Psi) \\ \Theta' = \dfrac{\varepsilon}{\Theta}\left[M - N\Theta + q(A\cos(\varphi+\Psi) - \sin\varphi)\cos\varphi\right] \end{cases} \qquad (3)$$

where prime denotes a derivation by the new variable ϕ.

To analyze resonance regimes one introduces the detuning parameter α for the rotor angle velocity Θ and the unit velocity of the elastic subsystem as $\Theta - 1 = \varepsilon\alpha$. Then the system (3) is averaged on the fast variable ϕ. One has, as a result, the system for the slow variables $A, \Psi, \Theta$. Stationary solutions are obtained from the next equations:

$$A = -K/2((\Theta-1)^2 + h^2/4)^{1/2},\ tg\Psi = -2(\Theta-1)/h,\ M - N\Theta = qhA^2/(2K) \qquad (4)$$

Checking numerical calculations show that the obtained analytical solutions resonance regimes with good accuracy. Moreover, the solutions is acceptable far from the resonance too. Let us the mechanical characteristic of the engine is the following: $L = K_L(\Omega_0 - \dot{\varphi})$. A dependence of the velocity Θ and the coefficient $K_L$, which defines a steepness of the characteristic, for the stationary regimes, is presented in the Fig. 2.

**Stability** of the stationary regimes is analyzed by using the corresponding variational equations. Stable regimes are presented in the Fig. 3. The interval $\Delta\Omega$ corresponds to unstable regimes, and cannot be realized in concrete mechanical systems.

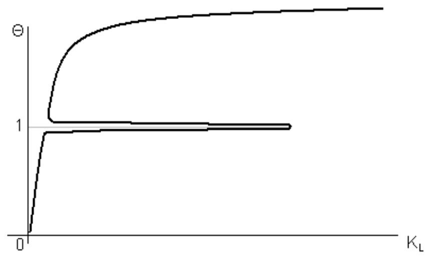



**Figure 2.** A dependence of the velocity Θ and the coefficient $K_L$ .

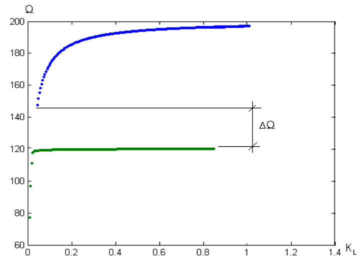



**Figure 3.** Stable regimes of motion

The engine frequency transfer when the parameter $K_L$ changes, for a case when the energy source is large, is shown in the Fig. 3.a But the Fig. 3.b shows the Zommerfeld effect, that is the engine "hovering" in the resonance regime, for a case when the energy source has a limited power supply, and the system is non-ideal.

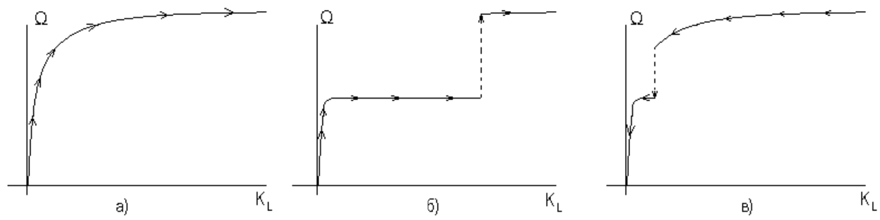



**Figure 4**. The engine frequency transfer. (a,c) the Zommerfeld effect is absent, (b) the Zommerfeld effect exists.

A procedure of the multiple-scale method [4] permits to describe a **trinsient** in the system (1). The numerical checking calculations show a good exactness of the transfer analytical presentation in a region of the non-resonance stationary regime stability. But if this stationary regime is unstable, on has a transfer to the 1:1 resonance stationary regime with large amplitudes. It is important that an exactness of the analytical presentation is good for sufficiently large time interval.

**Absorption of the resonance regimes.** Extinguishing of the resonance regime can be made by using the nonlinear absorber (Fig.1). Equations of motion in this case are the following:

$$
\begin{aligned}
&I\ddot{\varphi} = L(\dot{\varphi}) - H(\dot{\varphi}) + c_1 r(x - r\sin(\varphi))\cos(\varphi) \\
&M\ddot{x} + \beta\dot{x} + cx + c_2(x-y) + \gamma(x-y)^3 = c_1 r\sin(\varphi) \\
&m\ddot{y} + \beta\dot{y} - c_2(x-y) - \gamma(x-y)^3 = 0
\end{aligned}
\qquad (5)
$$

The nonlinear normal vibration modes conception can be used here. We can obtain the vibration mode which is appropriate for the absorption, when amplitudes of the main elastic subsystem are small, and amplitudes of the absorber are large. In this case the vibration energy localizes in the absorber. We can obtain too the vibration mode which is not appropriate for the absorption, when amplitudes of the main elastic subsystem are not small. Investigation of the modes stability permits to find values of the system parameters when such absorption takes place.

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