Nonlinear Dissipative Systems in Vicinity of Internal and Forced Resonances

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Summary. Free and forced dynamics of some nonlinear dissipative systems in vicinity of internal resonance is considered. A reduced system with respect to the system energy, an arctangent of the vibration amplitudes ratio, and the phase difference is used in the analysis.

Introduction

Investigation of behavior of nonlinear systems near internal resonance is an important step to solve numerous theoretical and applied problems. It means, in particular, problems of energy transfer and localization [1-4]. The internal resonance can lead to a loss of stability of vibration modes, and to appearance of new vibration regimes as a result of bifurcation [1-4]. Dissipation in nonlinear system can lead the system under consideration to the internal resonance, or to output the system from the resonance region. Here two 2-DOF nonlinear dissipative elastic systems (Figs. 1 and 2) are considered in a vicinity of internal resonance. An analysis is made by using so-called *reduced system* [5] which is written with respect to the system total energy, an arctangent of the ratio of amplitudes and a difference of phases. Investigation of stability and bifurcation of vibration modes which are similar to nonlinear normal modes (NNMs) [1,2,6] is made. In dissipative systems such regimes will contain an exponential decrease of the vibration amplitudes.

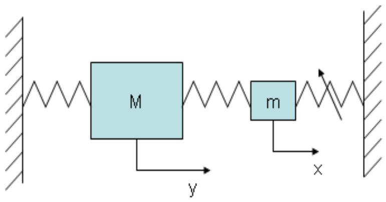

Fig. 1 – The spring-mass system

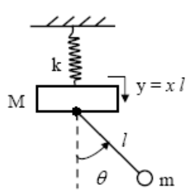

Fig. 2 – The spring-mass-pendulum system

Resonance behavior of the nonlinear spring-mass system

The spring-mass system (Fig.1) is considered in assumption that the mass m is essentially smaller than the mass M ; the anchor spring is nonlinear of the Duffing-type. Corresponding small parameters are introduced.

There are two NNMs in the system without dissipation: the non-localized mode of coupled vibrations, when amplitudes of both masses are compared, and the localized mode, when amplitudes of the small mass are essentially larger than ones of the mass M . The multiple scales method [7] is used. The *reduced system* with respect to the system total energy K , an arctangent of the ratio of amplitudes ψ and a difference of phases φ, can be written as the following:

$$\begin{cases} K' = (-\frac{L}{R}\cos^2\psi + \frac{D}{F}\sin^2\psi)K - K^3\cos^2\psi\sin^2\psi[\frac{I}{F}\sin(2\varphi) - \frac{E}{F}\cos(2\varphi)] \\ \psi' = (\frac{L}{R} + \frac{D}{F})\cos\psi\sin\psi - K^2\sin\psi\cos^3\psi[\frac{I}{F}\sin(2\varphi) - \frac{E}{F}\cos(2\varphi)] \\ \varphi' = -\frac{S}{R} + \frac{P}{F} - K^2\cos^2\psi[\frac{Q}{F} + \frac{I}{F}\cos(2\varphi) + \frac{E}{F}\sin(2\varphi)], \end{cases} \quad (1)$$

where all coefficients are determined by parameters of the system under consideration and of the detuning parameter Δ.

Analysis of the reduced system (1) equilibrium points shows that the coupled vibration mode loses stability in a vicinity of resonance, and the localized mode is stable for all initial conditions and the system parameters. It is obtained that there is a transfer from the non-localized mode to localized one at $t \to \infty$. New vibration modes do not appear.

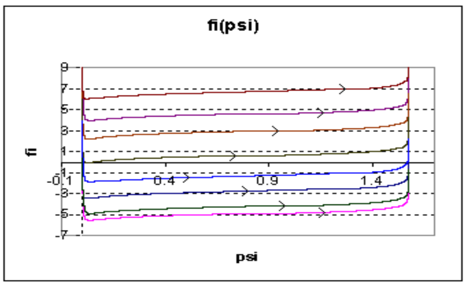

Fig. 3. Dependence $\varphi(\psi)$.

In Figure 3 the dependence $\varphi(\psi)$ is represented. The straight line $\psi = 0$ corresponds to the non-localized mode of connected vibrations, and the straight line $\psi = \pi/2$ corresponds to the mode when the energy is localized on the coordinate x. Trajectories of motions in the system configuration space correspond to the obtained analytical results.

Resonance behavior of the spring-mass-pendulum system

There are two nonlinear normal modes in the spring-mass-pendulum system without dissipation: the x-mode of vertical vibrations ($x = x(t)$, $\theta = 0$) which is localized, and the non-localized pendulum mode ($x = x(t)$, $\theta = \theta(t)$) when both vibration amplitudes are of the same order. Transfer to the reduced system, which is similar to the system (1) is used. Analysis of the obtained reduced system on equilibrium points shows that depending on energy level of the system it can obtain a region where vertical vibrations lose stability as a result of bifurcation. A transition to two modes of the coupled vibrations is realized. Then, when the energy decreases, there is an outcome from this region, the bifurcation disappears, and the vertical vibration mode again becomes stable. In Figures 4,5 a dependence $\varphi(\psi)$ for a case when the system is in region of existence of bifurcation, and for a case when the system is not in this region, respectively, are presented. The straight line $\psi = 0$ corresponds to the localized vibrations.

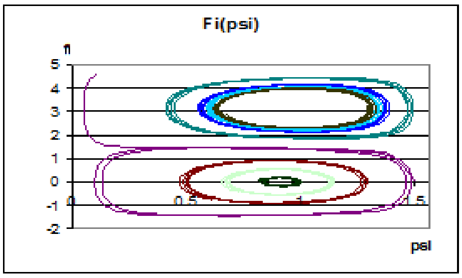

Fig. 4. Dependence $\varphi(\psi)$. The bifurcation of the vertical vibrations exist.

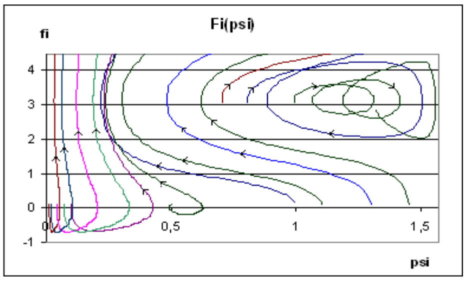

Fig. 5. Dependence $\varphi(\psi)$. The bifurcation of the vertical vibrations does not exist.

Trajectories of motions in the system configuration space correspond to the obtained analytical results.
Additional analysis of stability of vertical vibrations is conducted. It is obtained that the vertical vibration mode stability depends on time that confirms results of previous analysis of the reduced system.

Forced vibrations of nonlinear dissipative system having the internal resonance

The presented above approach can be applied in analysis of forced dynamics of the dissipative system which contains the nonlinear absorber (Fig. 6). It is made a detailed analysis of the system behavior in vicinity of the resonances on two fundamental frequencies, and in a case when both external and internal resonances are realized. In particular, the *transient nonlinear normal modes* of the dissipative system, which are realized only for some levels of the dissipative system energy, are observed. In vicinity of time, corresponding to the energy value, the system motions are close to these vibration modes.

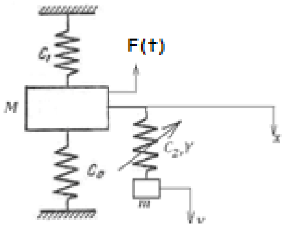

Fig.6. The mechanical system containing the nonlinear absorber.

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