Forced Nonlinear Normal Modes in the One Disk Rotor Dynamics

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Summary. A new approach combining both the nonlinear normal modes approach and the Rauscher method is proposed to construct forced vibrations in non-autonomous systems with an internal resonance. Forced vibrations of a one-disk unbalanced rotor with the nonlinear elastic bearings are considered. Gyroscopic effects, an asymmetrical disposition of the disk in the isotropic elastic shaft and internal resonance are taken into account.

Introduction

Nonlinear normal vibrations modes (NNMs) are a generalization of the normal vibrations in linear systems. The Shaw and Pierre concept of NNMs [1-3] is based on computation of invariant manifolds of motion on which the NNMs take place. By the Shaw-Pierre concept, the NNM is such a regime when all generalized coordinates and velocities are univalent functions of the selected couple of so-called "master" (or active) phase variables (*u, v*), where *u* is some dominant generalized coordinate, and *v* is the corresponding generalized velocity. The master coordinates can be chosen as new independent ones instead of time. In the case of internal resonance *four phase coordinates are active*, and they must be chosen as new independent variables. General procedures of the Rauscher method [4] utilization to construct NNMs in n-DOF non-autonomous systems are described in [3,5,6]. For the case of both external and internal resonances, the Shaw-Pierre approach together with the Rauscher method permits to reduce the n-DOF non-autonomous dynamical system to a two-degree-of-freedom nonlinear system for each NNM of forced vibrations. Analysis of nonlinear effects in the dynamics of the rotor systems can be found in different publications, in particular, in [7-10]. The forced NNMs are constructed here for the rotor system with the internal resonance by means of the NNM approach and the modified Rauscher method.

Iterative procedure to construct forced NNMs in a case of internal resonance

One considers a nonlinear system under an external periodical excitation. It is assumed that two linearized frequencies ν_1 and ν_2 are close, and they are close to the external frequency, Ω. In this case two active generalized coordinates, $q_{1,2}$, and two corresponding velocities, $s_{1,2}$, may be taken as independent master coordinates to construct the forced NNM. One uses a representation of the active coordinates in the form of the Fourier series for a case when the other coordinates are essentially smaller than the active ones:

$$q_1 = \sum_k \left(A_k \cos k\Omega t + B_k \sin k\Omega t \right);\ s_1 = \Omega \sum_k \left(kB_k \cos k\Omega t - kA_k \sin k\Omega t \right);\ q_2 = ...;\ s_2 = ...; \qquad (1)$$

One has from here, using some trigonometric transformations that

$$\cos(\Omega t) = \alpha_1 q_1 + \alpha_2 s_1 + \alpha_2 q_2 + \alpha_3 s_2 + \alpha_5 q_1^2 + \alpha_6 s_1^2 + ... \qquad (2)$$

By using this relation, an n-DOF "pseudo-autonomous system" can be obtained instead of the initial non-autonomous system. It corresponds to the principal idea of the Rauscher method. In the "pseudo-autonomous system" system the NNMs are constructed in a power series by the selected phase coordinates. It permits to reduce the n-DOF system to the two-DOF one. Four master phase coordinates are obtained from this system as a Fourier series instead of the series (2). Then the iteration process is constructed.

Construction of the forced NNMs in the one-disk rotor system

A model of the rotor dynamics with an asymmetrical disposition of the disk in the shaft is considered (Fig.1). Gyroscopic effects, nonlinear flexible base and internal resonance are taken into account. Fixed and moving coordinate systems and positional angles are shown in the Fig. 2.

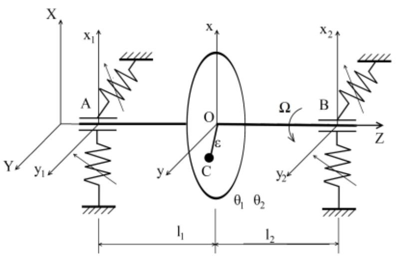

Fig.1. Principal model of the rotor system

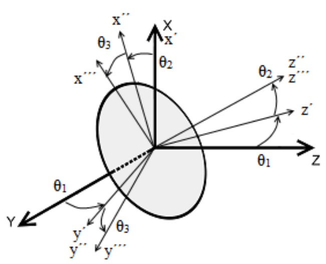

Fig. 2. Fixed and moving coordinate systems.

Equations of the rotor motion [10,11], which are not presented here, can be written as eight nonlinear ODEs describing the displacements and rotations of the disc and displacements at the supports, where the cubic nonlinearity in the restoring forces is considered. The procedure presented in Section 1 is used. Frequency responses of the generalized coordinates near the first resonance and trajectories of the resonance vibrations in the system configuration space are obtained. Checking numerical simulation and use of the harmonic balance method show a good exactness of the proposed procedure.

Analysis of the forced NNMs stability, which is made by calculation of multiplicators, shows that in some frequency range the resonance regimes having the cyclic symmetry trajectories may become unstable. In this region a pair of new solutions having center symmetry trajectories bifurcates. The frequency response for the first harmonic of the disk displacement x is shown in Fig. 3.

Additional analysis shows that an accounting of the dissipation in the shaft material by the Voigt hypothesis in the equations of motion leads to a narrowing of the region where the solutions having the center symmetry trajectories exist.

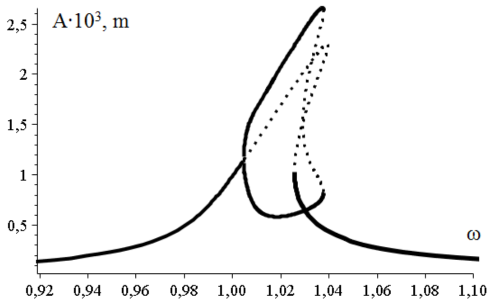

Fig. 3. Frequency response for the first harmonic of the disk displacement x. Results of the stability analysis are presented (solid lines correspond to stable solutions and points correspond to unstable ones).

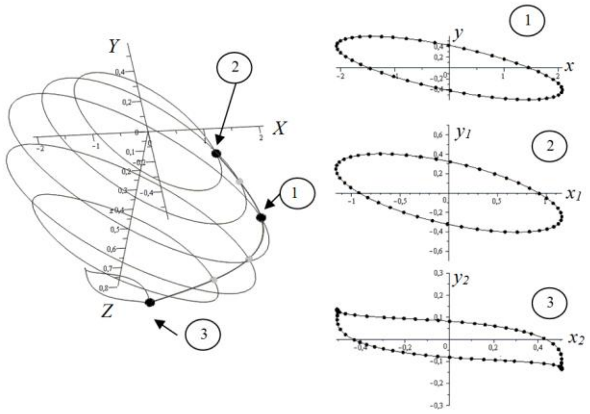

Fig. 4. Spacial representation of the rotor precession, corresponding to the regime with the center symmetry trajectories. The trajectories describe a motion of the disk center, left and right supports (points 1, 2, 3 respectively). The points correspond to the analytical solution; and the lines correspond to checking numerical calculations. All displacements are measured in mm.

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