FACULTY OF MECHANICAL ENGINEERING
UNIVERSITY OF WEST BOHEMIA

DEPARTMENT OF POWER SYSTEM ENGINEERING

Brakes, Thermal and Thermoelastic Analysis

Josef Voldřich
Ukraine, November 2018

Проект „Развитие международного сотрудничества с украинскими ВУЗами в областях качества, энергетики и транспорта“
г. Харьков, 11/2018

Степан Прокопович Тимошенко

1878 - 1972

- A grand native of Ukraine, the father of modern engineering mechanics.

- His portrait, as the only one, has displayed with reverence in my workroom for 20 years.

- He was a giant for mathematical modelling of strength problems in mechanical engineering.

Encouragement:

- Let's not be afraid to use MATHEMATICS for solution of our actual problems.
- And it is not inevitable to look up only to commercial FEM software.

Content

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Motivation and introduction

- Some problems with friction brakes

Brake thermal analysis

- Modeling of friction element contacts
- Contact surface temperature – analytical approach
- Finite elements models

Uncoupled thermal stress and distortion analysis

Coupled thermal and distortion analysis

The last three parts of the lecture is above all from

J. Voldřich: Brakes, thermal and thermoelastic analysis. In: Hetnarski R. (Ed.), *Encyclopedia of Thermal Stresses*, Vol 1, pp 486-497, Springer Dordrecht, Heidelberg, New York, London 2014

Motivation and introduction

Motivation

ŠKODA

- The springtime 2002 – the chief of undercarriage development department of Škoda Auto, Ing. J. Nepomucký CSc., came to with a problem concerning „**hot judder**“ (hot vibration) in disk brakes.
- Nobody of the whole concern Volkswagen AG knew a real cause of the phenomenon and any way how to eliminate it. It was a reason to deal with breaks.

(*Volkswagen Group* = $\sum Volkswagen, Škoda\ Auto, Audi, Porsche, SEAT, \ldots$)

- Problems with hot judder and brake fading appeared even in the German car factory Volkswagen AG.

Motivation and introduction

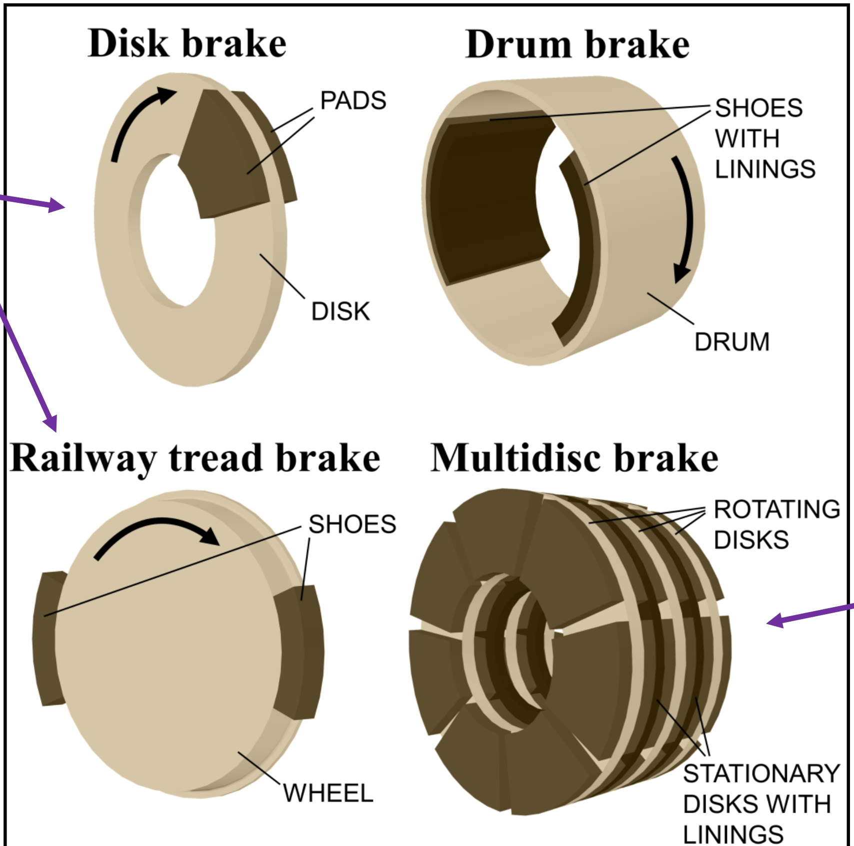
A mechanical friction brake is a technical device that serves to slow or stop a moving body, or for keeping it at rest.

The main components of the friction brakes are friction elements.

In braking, a rotating body or system of bodies are in contact with fixed friction elements.

Intermittent contact

The brakes are classified with respect to the arrangement of the brake elements.



Full contact

Motivation and introduction

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Examples of disk brakes

http://en.wikipedia.org/wiki/Disc_brake

Mercedes Benz AMG carbon ceramic brake

Motivation and introduction

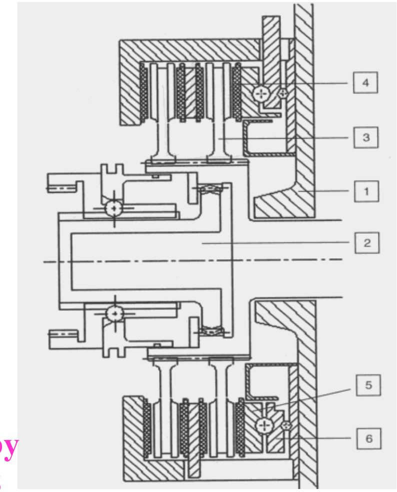
wikipedia

Aircraft brakes

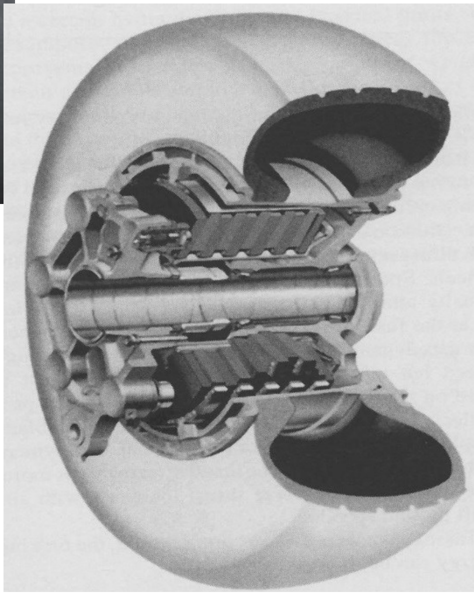
The energy dissipated by the disk brake when stopping a passenger car weighting 1,500 kg from a speed of 100 km/h makes 0.15 MJ.

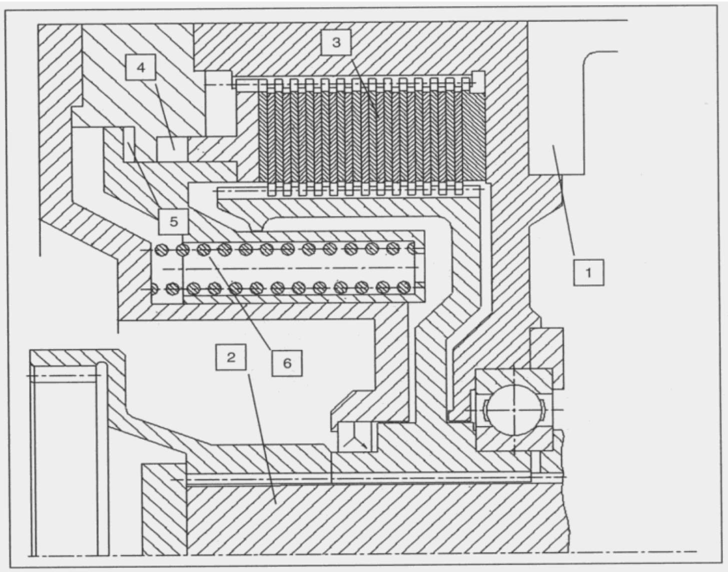
Each 10-disk brake of a Boeing 777 passenger aircraft must be capable of absorbing up to 144 MJ.

Multidisk brakes



out of the book by Breuer, Bill 2008





Why passenger aircrafts are not able to take off immediately after their landing?

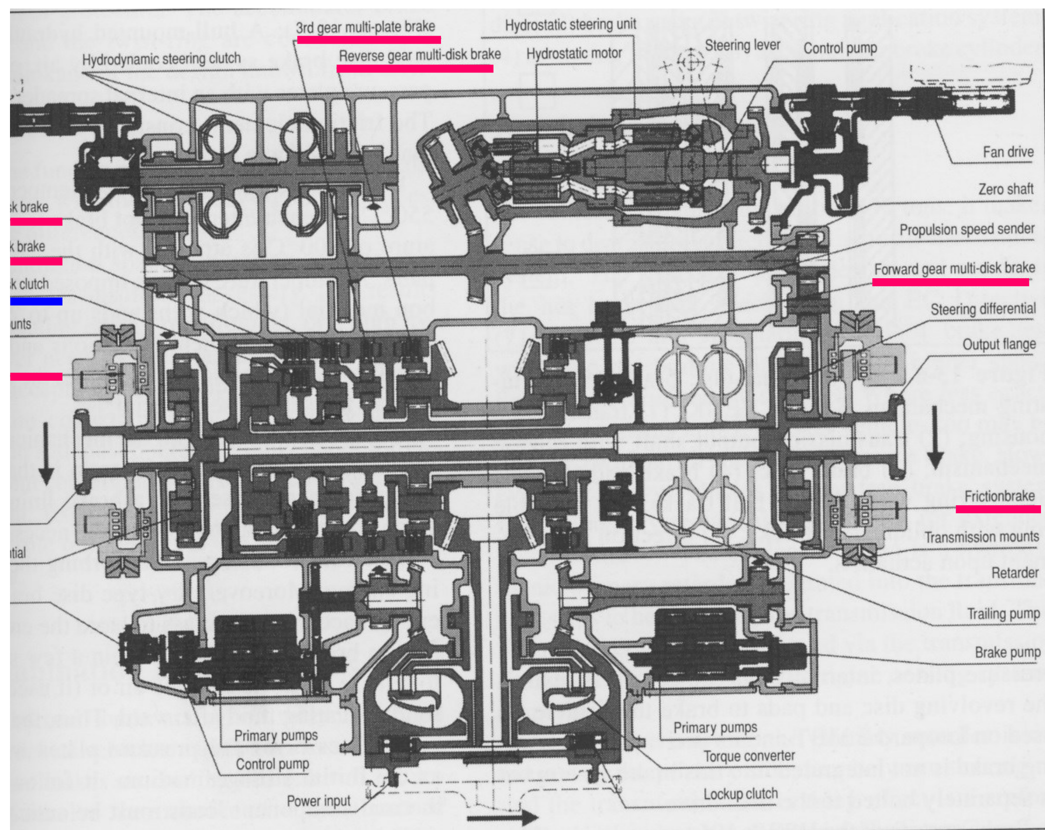
Hot brakes are one of the main reason.

Motivation and introduction

Leopard 2 during all-out braking operation.

Schematic view of the 1100kW transmission HSWL 354 for heavy tracked vehicles weighing more than 60 tons.

Red lines label brakes.



Motivation and introduction

Some problems with brakes

Thermal stability

Fading – generally a drop of braking power and braking eefect at high temperatures

Formation of bubbles due to evaporation – the brake fluid reaching boiling temperature at the hottest point in the brake caliper

Brake disk deflection – inadequate thermal stability may result from a geometric error

Brake noise

- very difficult to forecast by way of calculation methods

Wear

Corrosion, material degradation

Cracks

Brake judder

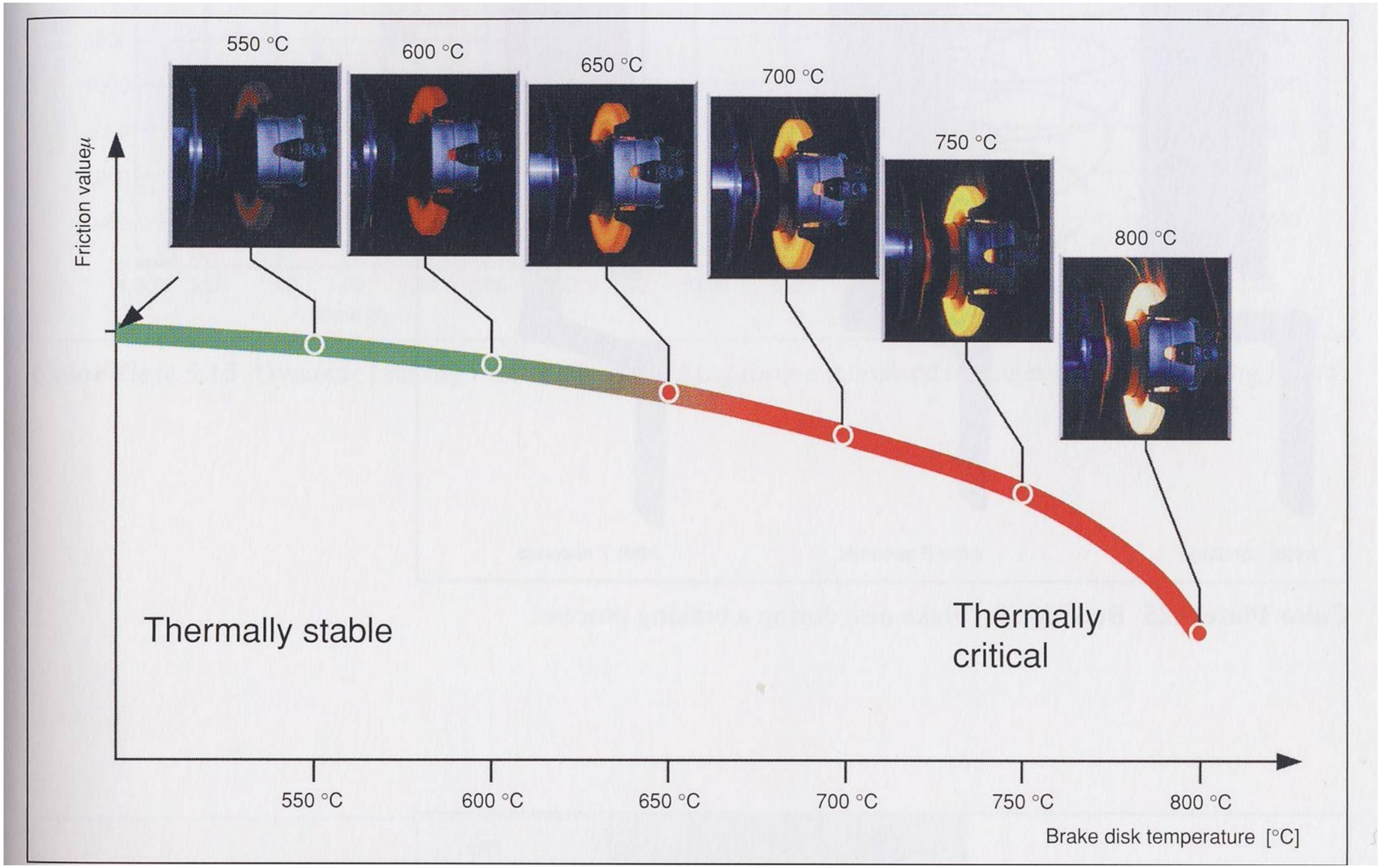
- hot judder is induced by thermoelastic instability (our next lecture)

Motivation and introduction

Brake fading

Current vehicular disk brake

Friction value



Brake disk temperature

Motivation and introduction

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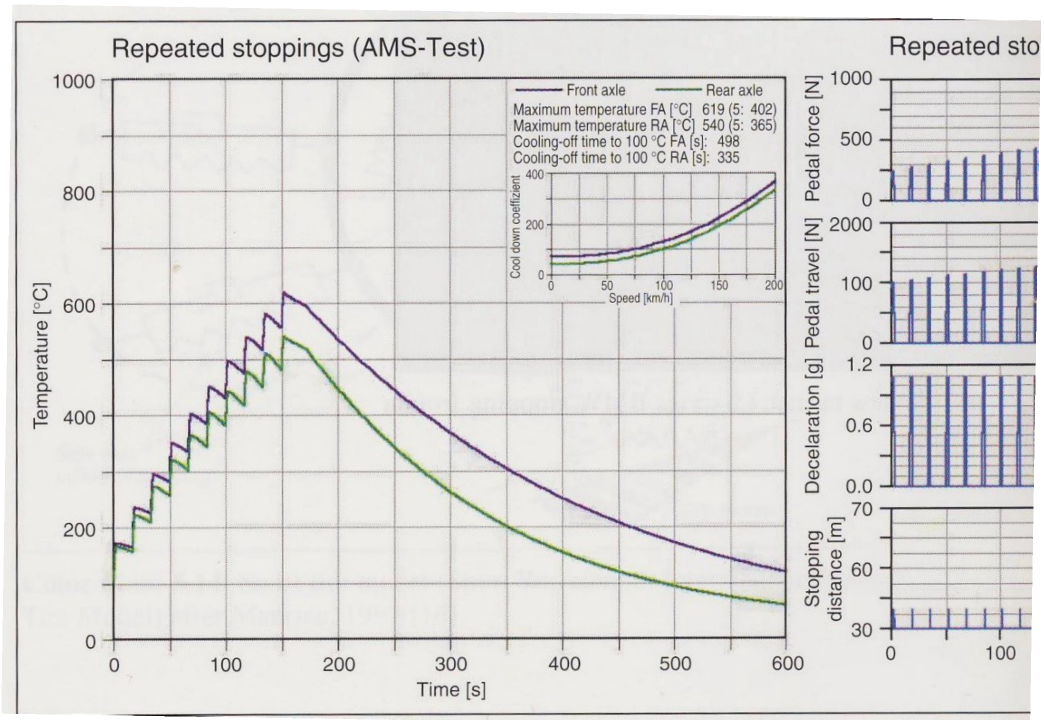
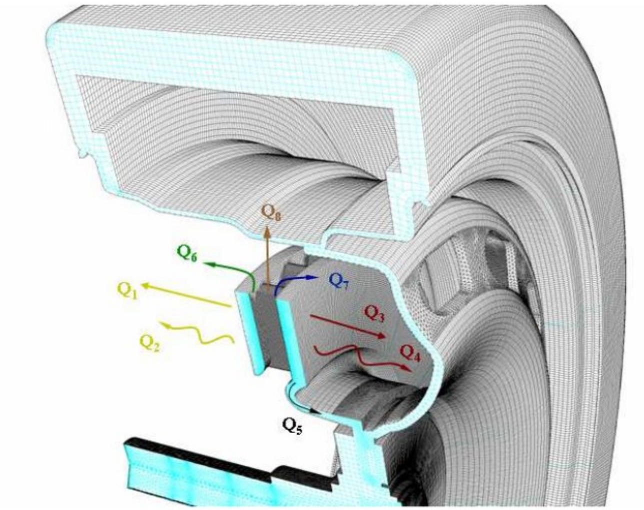
Repeated stoppings test

Temperature



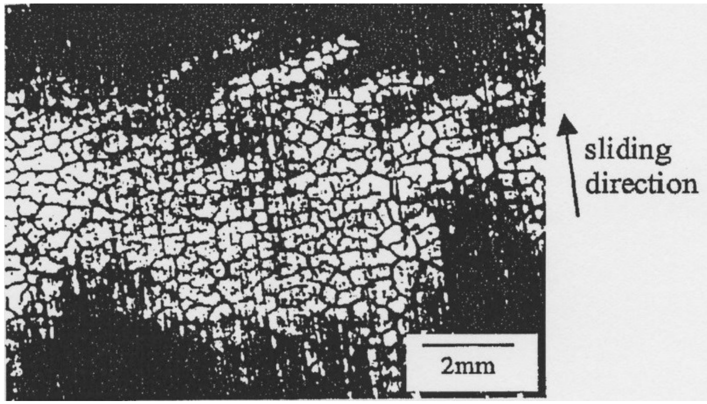
Time

Brake system cooling



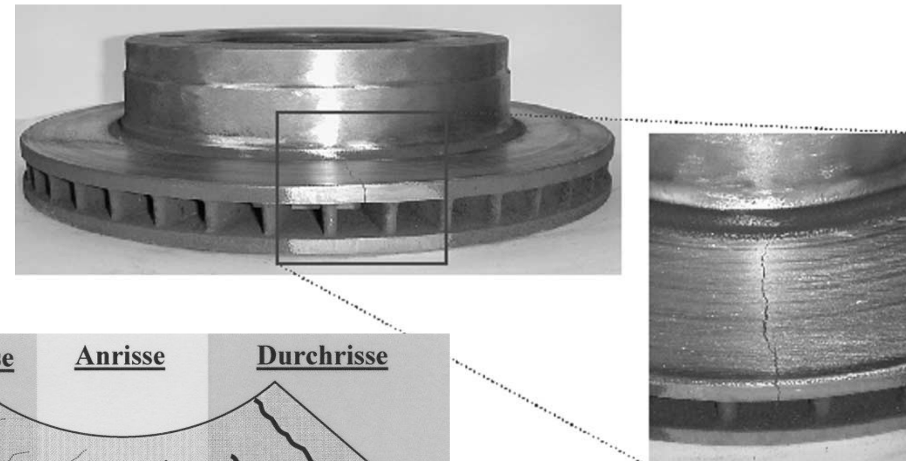
Motivation and introduction

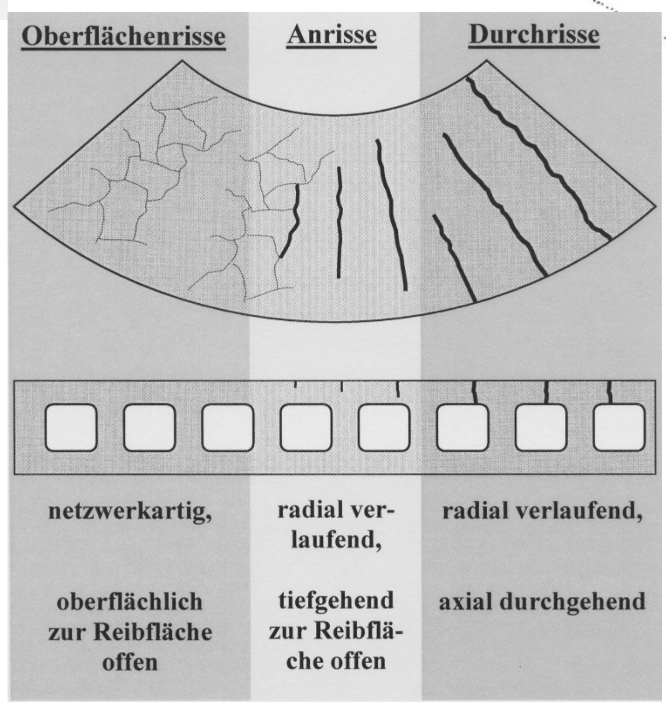
Cracks



Dufrénoy, Weichert, *J Thermal Stresses* 26(2003), 815-828

Nguyen-Tajan 2002 -disertace





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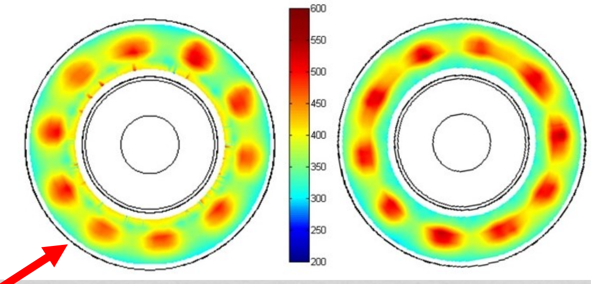
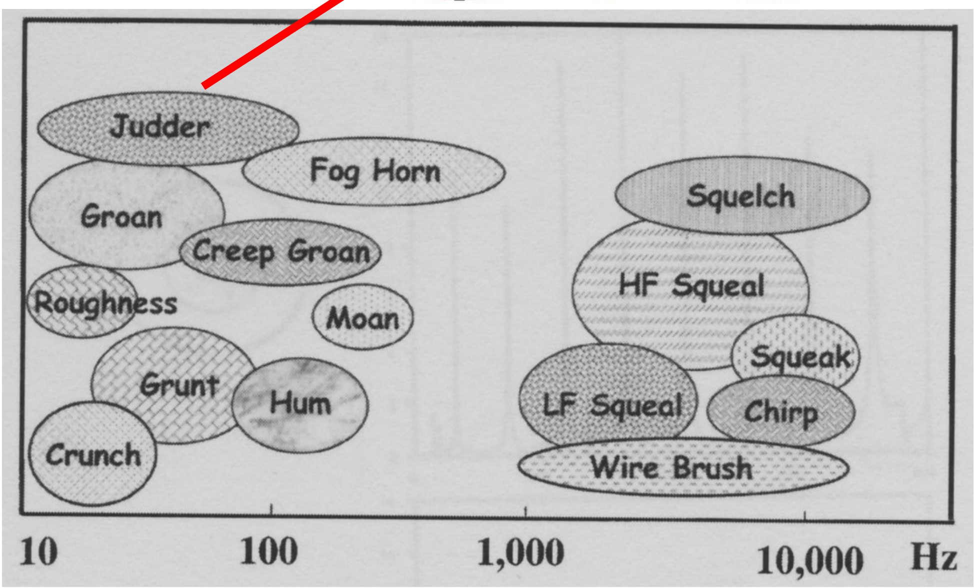
Motivation and introduction

Hot judder



Hot spots appear on both sides of disk when hot judder occurs.

measured at our University of West Bohemia



A. Akay: Acoustics of friction, *J. Acoust. Soc. Am.* 111 (2002), 1525-48

Prof. Barber 1969 described the frictionally excited thermoelastic instability as the cause of the phenomenon that is of critical importance in the design of brakes and clutches.

Brake thermal analysis

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Modeling of friction element contacts

From a microscopic point of view, contact with friction between two bodies 1 and 2 is a very complex effect, which is affected by the surface roughness, composition of the materials used, their wear, ...

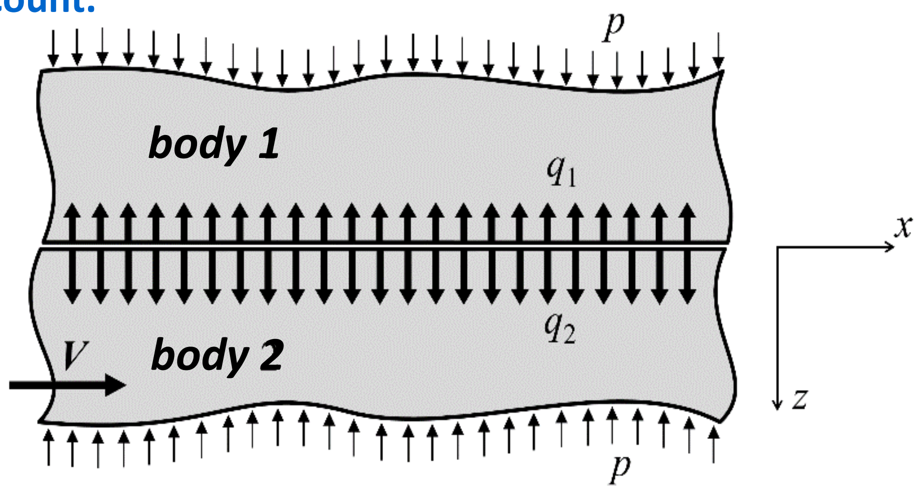
We take only macroscopic physical entities into account.

$$T_1(x, y, 0, t) = T_2(x, y, 0, t)$$

$$q = f\,pV$$

$$q = q_1 + q_2$$

$$q_1 = K_1 \frac{\partial T_1}{\partial z} \qquad q_2 = -K_2 \frac{\partial T_2}{\partial z}$$



T_i – temperature of body i
f – friction coefficient
V – sliding velocity
p – normal contact pressure

q – the heat produced by bodies friction per time unit applied to a unit area
q_i – the heat flux removed from the contact surface to the body i

K_i – the thermal conductivity coefficient of body i

Brake thermal analysis

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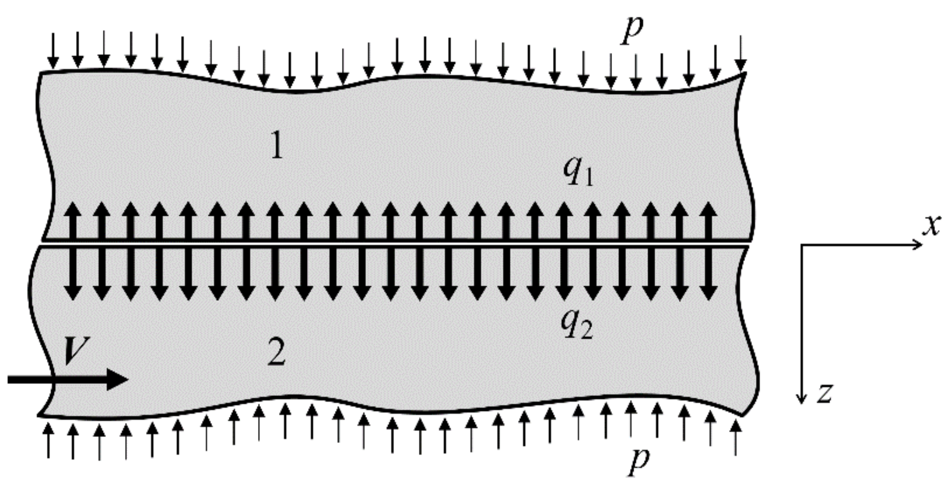
Modeling of friction element contacts



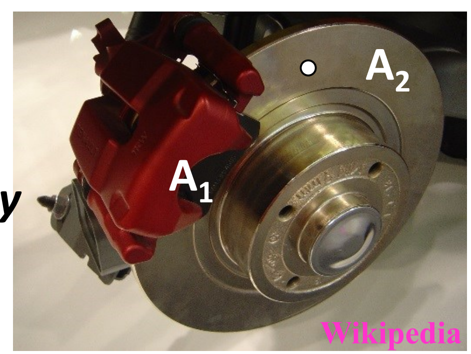
Full contact

$$q_1 = q\left(1 + \frac{K_2}{K_1}\sqrt{\frac{k_1}{k_2}}\right)^{-1},$$

$$q_2 = q\left(1 + \frac{K_1}{K_2}\sqrt{\frac{k_2}{k_1}}\right)^{-1},$$

K_i* – the thermal conductivity coefficient of body *i

***$k_i = K_i/c\rho$* – the thermal diffusivity**



Intermittent contact

$$q_1 = q\left(1 + \frac{A_2}{A_1}\frac{K_2}{K_1}\sqrt{\frac{k_1}{k_2}}\right)^{-1},$$

$$q_2 = q\left(1 + \frac{A_1}{A_2}\frac{K_1}{K_2}\sqrt{\frac{k_2}{k_1}}\right)^{-1},$$

A_i* – the size of the corresponding contact area of body *i

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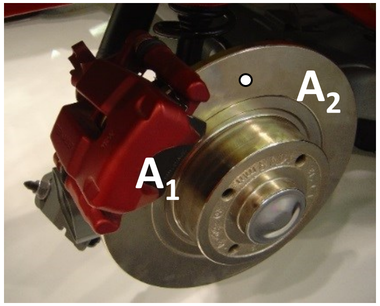
Example

A contemporary automobile disk brake

- *disk from cast iron*
- *the friction material A of pads*
- *at least $A_2/A_1 = 7$*

... approximately 98% of the produced heat goes into the disk !!

Intermittent contact



$$q_1 = q\left(1 + \frac{A_2}{A_1}\frac{K_2}{K_1}\sqrt{\frac{k_1}{k_2}}\right)^{-1},$$

$$q_2 = q\left(1 + \frac{A_1}{A_2}\frac{K_1}{K_2}\sqrt{\frac{k_2}{k_1}}\right)^{-1},$$

Brakes, Thermal and Thermoelastic Analysis,
Table 1 Orientation values of parameters of some materials in use. k – thermal conductivity, κ – thermal diffusivity, E – elastic modulus, α – coefficient of thermal expansion, ρ – density

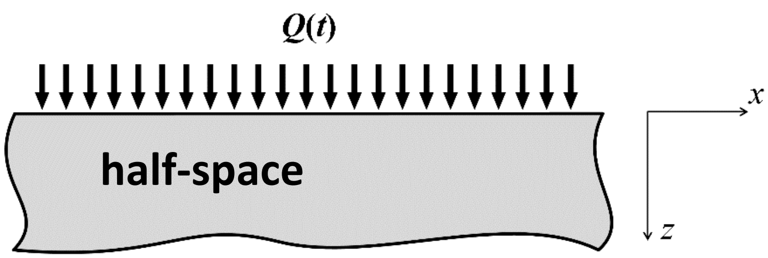
	k W/(mK)	$\kappa.10^6$ m^2/s	E GPa	$\alpha.10^6$ 1/K	ρ kg/m^3
Cast iron	54	12.98	125	12	7,100
Friction material A	5	3.57	1	10	4,000
Friction material B	1.2	0.52	8	15	3,000
Composite C/SiC	40	22.2	30	0.5	2,300

Brake thermal analysis

Carslaw, Jaeger 1959

Contact surface temperature

Analytical solution for the heated half-space - 1D problem



We receive, using Laplace transform

for $Q(t) \equiv Q = 1$

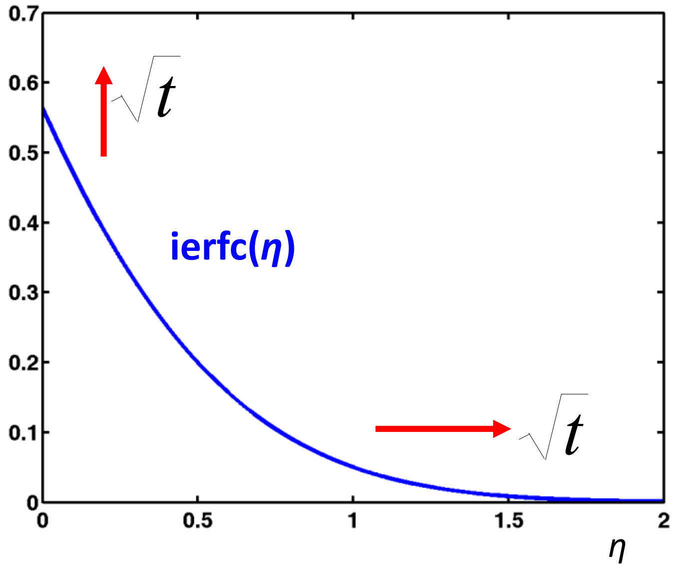
$$\Theta_I(z,t) = \Theta_0 + \frac{2\sqrt{k\,t}}{K}\,\mathrm{ierfc}\left(\frac{z}{2\sqrt{k\,t}}\right), \quad z \geq 0, \quad t > 0,$$

where

$$\mathrm{ierfc}(\eta) = \frac{1}{\sqrt{\pi}}\exp(-\eta^2) - \eta\,(1-\mathrm{erf}(\eta)),$$

$$\mathrm{erf}(\eta) = \frac{2}{\sqrt{\pi}}\int_0^{\eta}\exp(-\zeta^2)\,\mathrm{d}\zeta$$

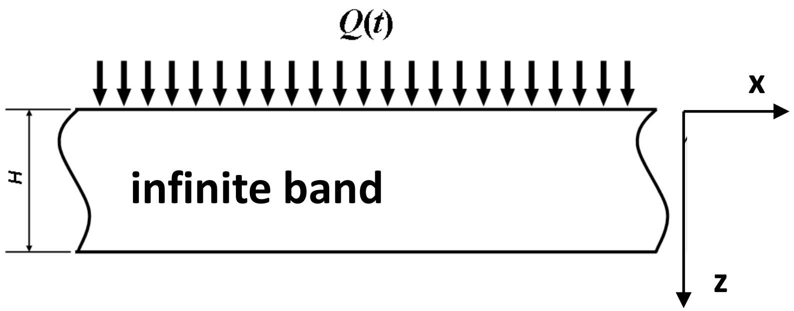
the error function



Surface temperature

$$Q\frac{2\sqrt{k\,t}}{K\sqrt{\pi}} = Q\frac{2\sqrt{t}}{\sqrt{K\rho\,c\pi}}$$

Brake thermal analysis



$$\frac{\partial T(z,t)}{\partial t} = k\frac{\partial^2 T(z,t)}{\partial z^2},\quad 0 \le z \le H,\ t > 0,$$

$$-K\frac{\partial T(H,t)}{\partial z} = 0,\quad K\frac{\partial T(0,t)}{\partial z} = Q(t),\quad T(z,0) = \Theta_0$$

Solution by variable separation method

very complicated, unsuitable formula

$$T(z,t) = \frac{Q(t)}{K}\left(\frac{z}{2H} - \frac{H}{6}\right) + \frac{k}{KH}\int_0^t Q(\tau)\,d\tau + Q(0)\frac{2H}{K\pi^2}\sum_{n=1}^{\infty}\left\{\frac{(-1)^{n+1}}{n^2}\exp(-k\lambda_n^2 t)\cos(\lambda_n)\right\} +$$

$$+\frac{2H}{K\pi^2}\sum_{n=1}^{\infty}\left\{\frac{(-1)^{n+1}}{n^2}\cos(\lambda_n z)\int_0^t \exp\left(-k\lambda_n^2(t-\tau)\right)\frac{dQ(\tau)}{d\tau}\,d\tau\right\}$$

where $\lambda_n = \frac{\pi n}{H}$ and $\Theta_0 = 0$

(Carslaw, Jaeger 1959 - solution only for $Q(t) \equiv Q_0$)

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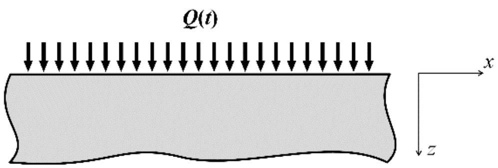
Solution by the Laplace transform



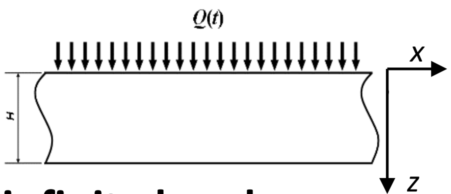
half-space

backward transformation

$$\frac{1}{s\sqrt{s}}\exp\left(-\alpha\sqrt{s}\right) \longrightarrow 2\sqrt{t}\ \mathrm{ierfc}\left(\frac{\alpha}{2\sqrt{t}}\right) \qquad \alpha = \frac{z}{\sqrt{k}}$$

for $Q(t) \equiv Q = 1$

$$\Theta_I(z,t) = \Theta_0 + \frac{2\sqrt{k\,t}}{K}\mathrm{ierfc}\left(\frac{z}{2\sqrt{k\,t}}\right), \quad z \geq 0, \quad t > 0,$$



infinite band

$$\frac{1}{s\sqrt{s}}\frac{e^{\alpha\sqrt{s}} + e^{(\beta-\alpha)\sqrt{s}}}{e^{\beta\sqrt{s}} - e^{-\beta\sqrt{s}}} \longrightarrow ?? \qquad \alpha = \frac{z}{\sqrt{k}}, \quad \beta = \frac{2H}{\sqrt{k}}$$

$$\frac{1}{s\sqrt{s}}\frac{e^{\alpha\sqrt{s}} + e^{(\beta-\alpha)\sqrt{s}}}{e^{\beta\sqrt{s}} - e^{-\beta\sqrt{s}}} = \frac{1}{s\sqrt{s}}\left(e^{(-\beta+\alpha)\sqrt{s}} + e^{-\alpha\sqrt{s}}\right)\left(1 + e^{-2\beta\sqrt{s}} + e^{-4\beta\sqrt{s}} + \ldots\right)$$

for $Q(t) \equiv Q = 1$

$$\Theta_{II}(z,t) = \Theta_0 + \frac{2\sqrt{kt}}{K}\sum_{n=0}^{\infty}\left\{\mathrm{ierfc}\left(\frac{2H(n+1)-z}{2\sqrt{kt}}\right) + \mathrm{ierfc}\left(\frac{2H\,n+z}{2\sqrt{kt}}\right)\right\}.$$

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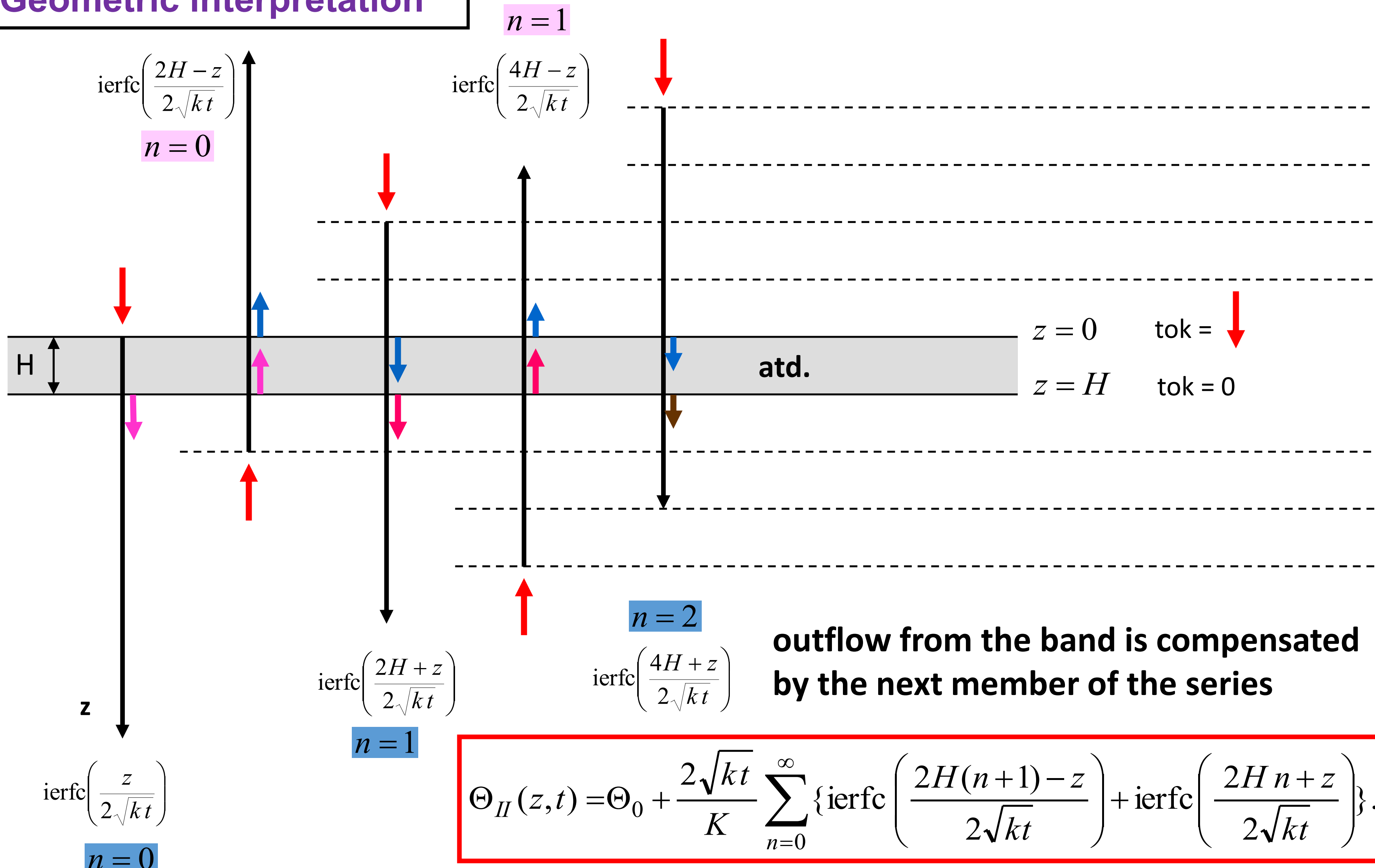
Geometric interpretation



outflow from the band is compensated by the next member of the series

$$\Theta_{II}(z,t) = \Theta_0 + \frac{2\sqrt{kt}}{K}\sum_{n=0}^{\infty}\left\{\mathrm{ierfc}\left(\frac{2H(n+1)-z}{2\sqrt{kt}}\right)+\mathrm{ierfc}\left(\frac{2H\,n+z}{2\sqrt{kt}}\right)\right\}.$$

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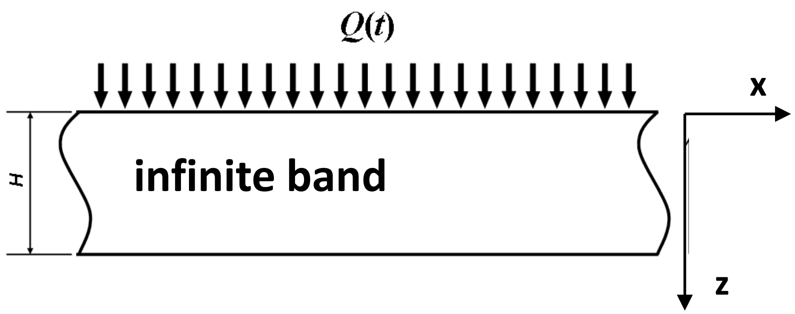
Time dependent heat flux



$$\frac{\partial T(z,t)}{\partial t} = k\frac{\partial^2 T(z,t)}{\partial z^2}, \quad 0 \le z \le H, \ t > 0,$$

$$-K\frac{\partial T(H,t)}{\partial z} = 0, \quad K\frac{\partial T(0,t)}{\partial z} = Q(t), \quad T(z,0) = \Theta_0 \qquad (*)$$

$$\Theta_{II}(z,t) = \Theta_0 + \frac{2\sqrt{kt}}{K}\sum_{n=0}^{\infty}\left\{\mathrm{ierfc}\left(\frac{2H(n+1)-z}{2\sqrt{kt}}\right) + \mathrm{ierfc}\left(\frac{2H\,n+z}{2\sqrt{kt}}\right)\right\}.$$

The general solution of the problem (*) using Duhamel's theorem

$$T(z,t) = \int_0^t \Theta_{II}(z,t-\tau)\frac{dQ(\tau)}{d\tau}d\tau + \sum_{j,\tau_j \le t_B}\Theta_{II}(z,t-\tau_j).\Delta Q(\tau_j)$$

if the function Q is smooth in the intervals (τ_j, τ_{j+1}) and has a jump point $\Delta Q(\tau_j) = Q^+(\tau_j) - Q^-(\tau_j)$ to the magnitude at the time points τ_j.

Brake thermal analysis

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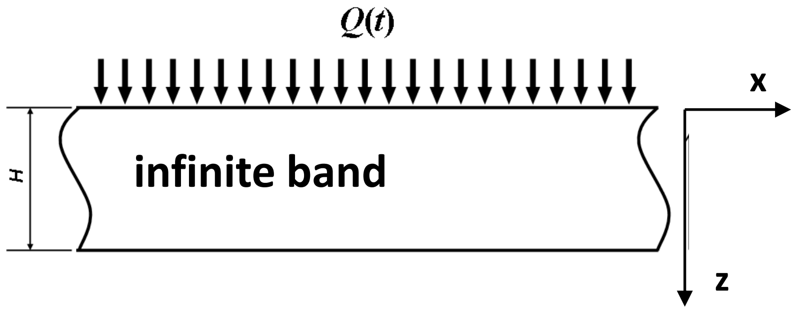
Full contact and emergency braking



V(t) = V₀.(1-t/t_B) – the sliding velocity

t_B - the braking time

$Q(t) = Q_0.(1-t/t_B)$ – both the contact pressure p and friction coefficient f are considered constant

approximation

$$\Theta_{II}(0,t) \approx \Theta_0 + \frac{2\sqrt{k}}{K\sqrt{\pi}}\sqrt{t}$$

$$T(z,t) = \int_0^t \Theta_{II}(z,t-\tau)\frac{dQ(\tau)}{d\tau}d\tau + \sum_{j,\tau_j \le t_B} \Theta_{II}(z,t-\tau_j).\Delta Q(\tau_j)$$

The first approximation of the surface temperature for emergency braking is **the Fazekas known formula** (1953)

$$T(0,t) \approx \Theta_0 + \frac{2q_0\sqrt{k}}{K\sqrt{\pi}}\sqrt{t\left(1-\frac{2t}{3t_B}\right)} \qquad 0 \le t \le t_B$$

Brake thermal analysis

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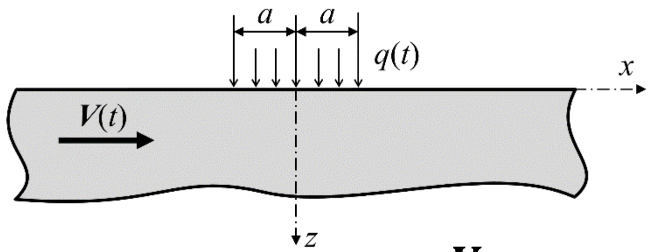
Intermittent contact and emergency braking



$V(t) = V_0.(1-t/t_B)$ ***– the sliding velocity***

t_B - the braking time

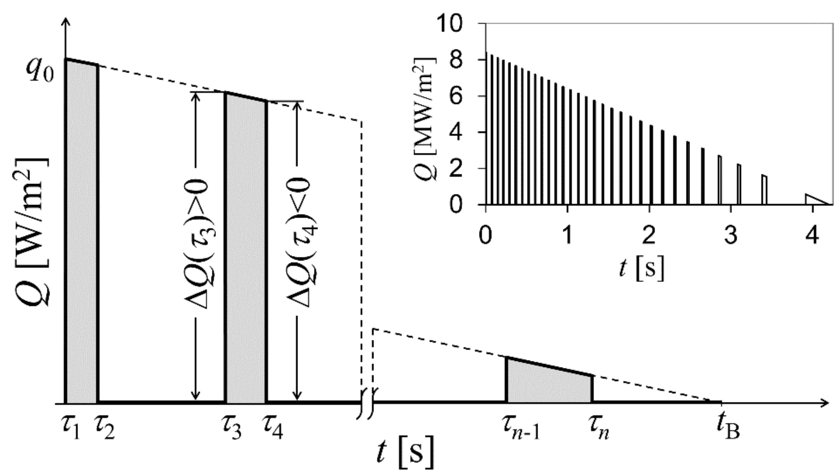
both the contact pressure p and friction coefficient f are considered constant

The Péclet number $\mathbf{Pe} = \dfrac{\boldsymbol{Va}}{\boldsymbol{2k}}$, where 2a is the full length of the contact (of the pad).

The one-dimensional approximation is useful for **Pe > 10**.

For intermittent contact, the flux $Q(t)$ is positive only if we consider passing a given contact point under the friction pad having 2*a* in width size.

***Q*(*t*)**

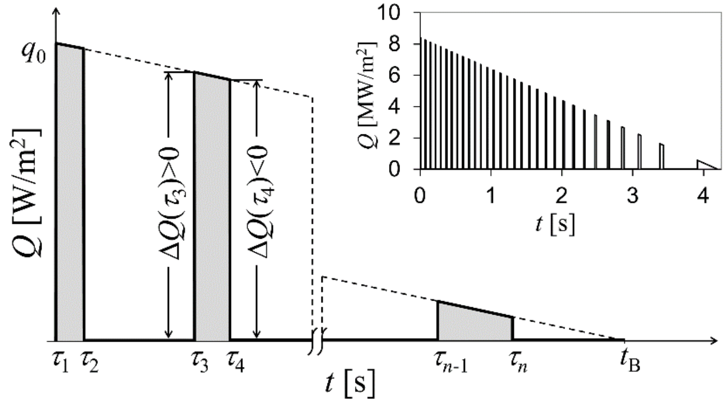


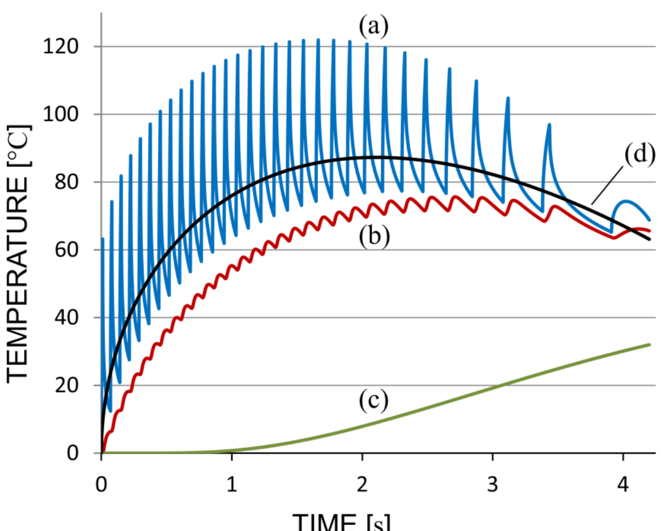
The rise $\boldsymbol{\Delta T}$ in temperature when the surface point is passing under the friction pad

$$\Delta T \approx q \frac{2\sqrt{k}}{K\sqrt{\pi}} \sqrt{\Delta t} = \frac{2qa}{K\sqrt{\pi}\sqrt{\mathrm{Pe}}}$$

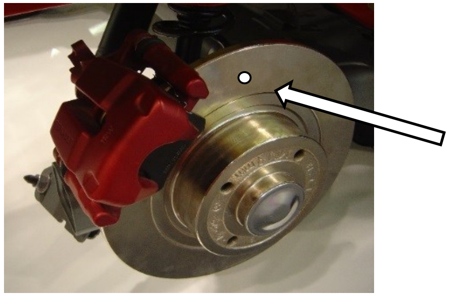
Brake thermal analysis

Example – emergency braking





(a) point at contact surface
(b) point 1.5 mm under surface
(c) point in the half thickness position of the disk
(d) mean surface temperature



a given point at disk surface

V_0 =11,2 m/s initial sliding velocity (which corresponds to the automobile velocity of 100 km/h)

t_B = 4,2 s braking time

q_0 = 8,4 W/mm² initial heat flux

$2a$ = 112 mm length of trajectory of a given point under the friction pad

L = 780 mm trajectory of the point in full revolution

H = 26,4 mm half thickness of the disk

Pe_0 = 24160 Péclet number for t=0

$$\Delta T \approx q \frac{2\sqrt{k}}{K\sqrt{\pi}}\sqrt{\Delta t} = \frac{2qa}{K\sqrt{\pi}\sqrt{\mathrm{Pe}}}$$

63,2°C

Brake thermal analysis

WARNING

about FEM calculations

Applied Thermal Engineering 31 (2011) 1003–1012

Contents lists available at ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng

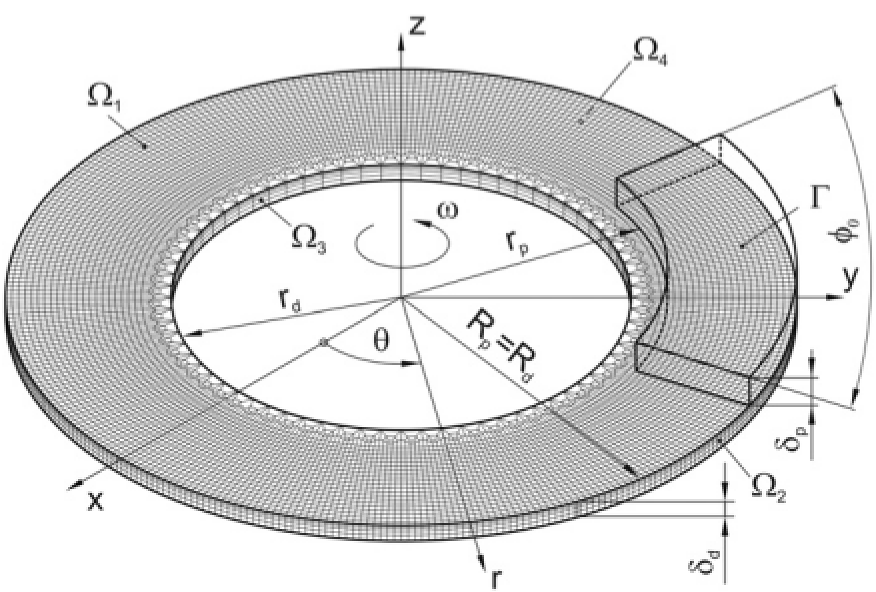
Analysis of disc brake temperature distribution during single braking under non-axisymmetric load

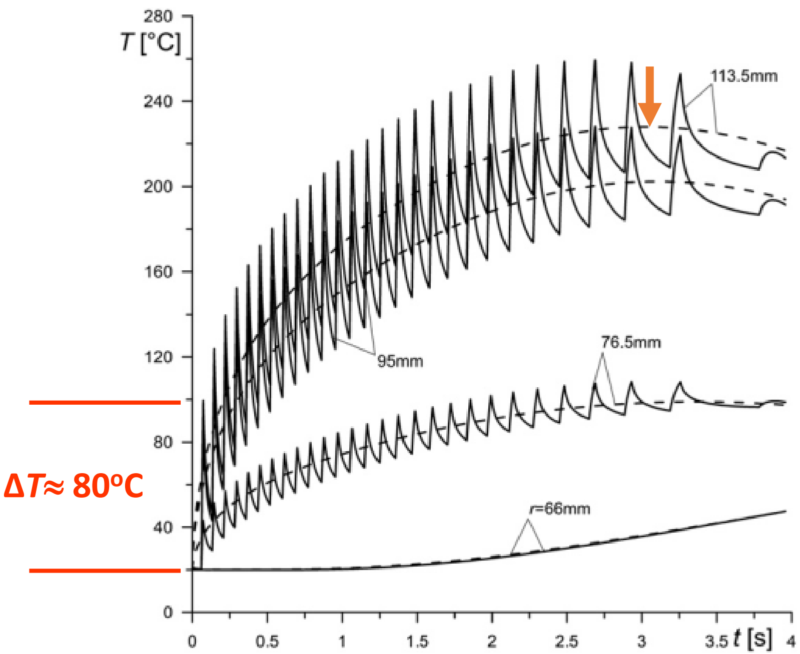
poslední impact faktor 1.823

[redacted]

five-year impact faktor 2.095

Faculty of Mechanical Engineering, Bialystok University of Technology (BUT), 45C Wiejska Street, Bialystok 15-351, Poland





analytical approach $\Delta T \approx 118^oC$, i.e. they have cca 32% error by FEM calculation

Brake thermal analysis

Maximal contact surface temperature

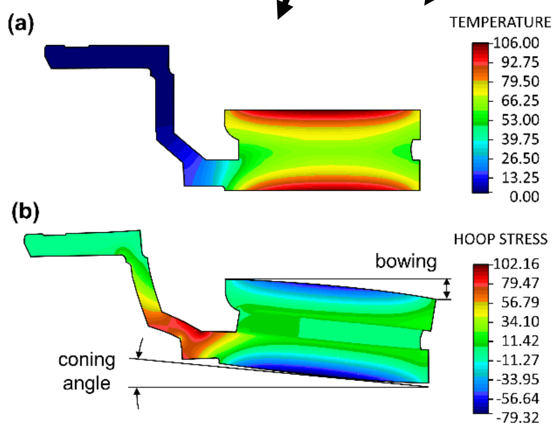
$$T_{max} = T_b \oplus T_s \oplus T_f \oplus T_{per}$$

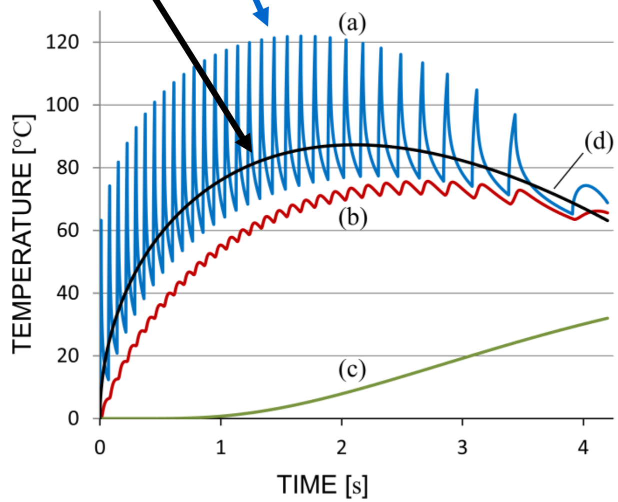
T_b – bulk temperature

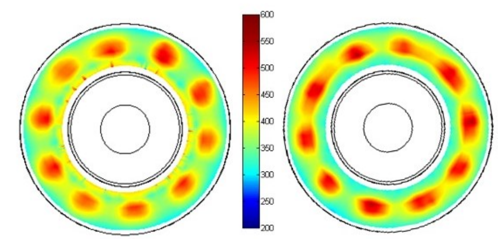
T_s – mean surface temperature

T_f – peak surface (flash) temperature

T_{per} – amplitude of temperature field perturbation (due to TEI)







ength and time measures, so that the
ferent degree of complexity, and
aches and computational efforts.

Typical are different length and time measures, so that analyses show a different degree of complexity and require different approaches and computational efforts !!

Brake thermal analysis

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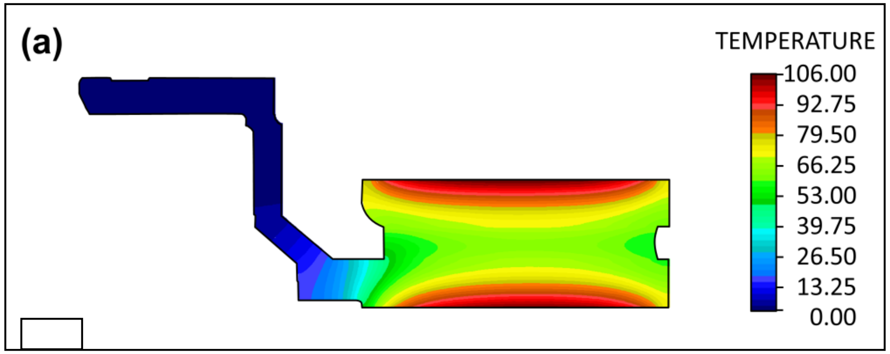
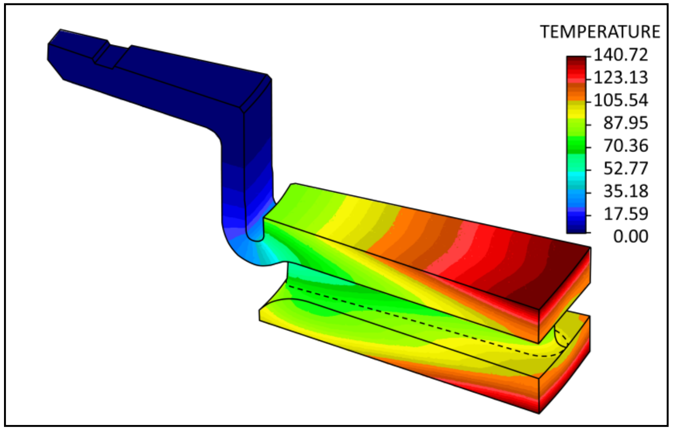
Standard FEM models are excellent for T_b and T_s

$$T_{max} = T_b \oplus T_s \oplus T_f \oplus T_{per}$$



2D axisymmetric model



3D model with periodicity

Uncoupled thermal stress

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Thermal stress - analytical approach (for flash temperature)

1D model and the stress acting on the thin surface layer

A particular solution to the thermoelastic equation in the form of a strain potential

$$2G\mathbf{u} = \nabla\Phi$$

where

$$\nabla^2\Phi = \frac{2G(1+\nu)\alpha}{1-\nu}T$$

$\mathbf{u}=(u_x,u_y,u_z)$ **displacement vector**

T **temperature field previously calculated**

α **coefficient of thermal expansion**

$G=\dfrac{E}{2(1+\nu)}$ **shear modulus**

T* and Φ are functions of only one space variation *z

$$\sigma_{xx} = \sigma_{yy} = \frac{\partial^2\Phi}{\partial z^2}$$

$$\sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0 \qquad \sigma_{zz} = 0$$

Brief heating of an intensity intenzity *q* over a time Δ*t* brings a compressive stress of the surface z=0

$$\bar{\sigma}_{xx} = \bar{\sigma}_{yy} \approx -\frac{2E\alpha\sqrt{k}}{(1-\nu)K\sqrt{\pi}}q\sqrt{\Delta t}$$

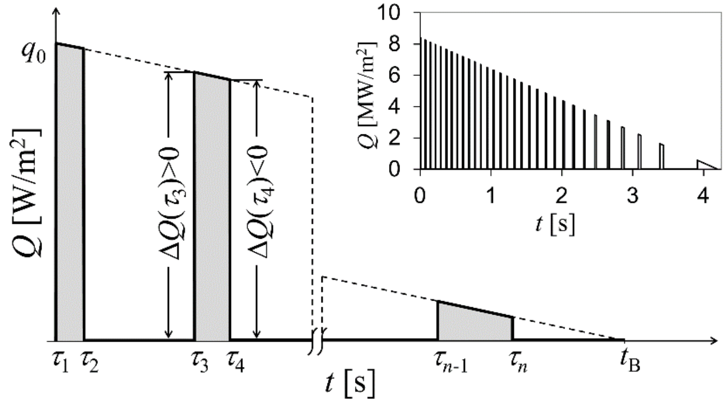
The difference of normal displacements of the surface under the pad (with a length 2*a*)

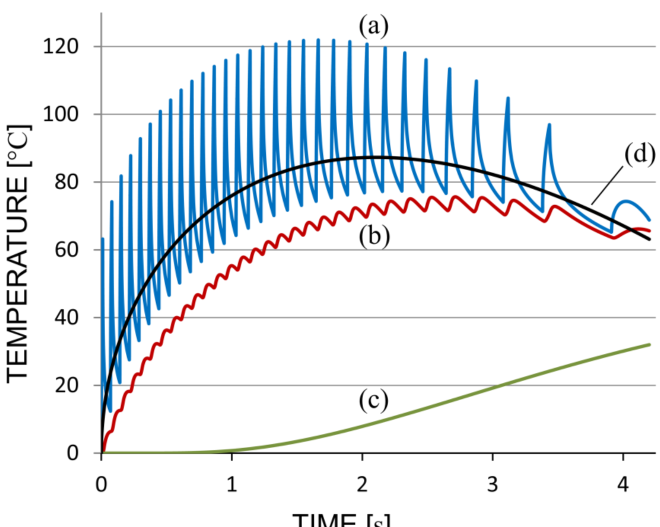
$$\Delta\bar{u}_z \approx -2c\,q\,a^2 / Pe$$

where $c = (1+\nu)\alpha / K$

Uncoupled thermal stress

Example – emergency braking





(a) point at contact surface
(b) point 1.5 mm under surface
(c) point in the half thickness position of the disk
(d) mean surface temperature

V_0 =11,2 m/s initial sliding velocity (which corresponds to the automobile velocity of 100 km/h)

t_B = 4,2 s braking time

q_0 = 8,4 W/mm² initial heat flux

$2a$ = 112 mm length of trajectory of a given point under the friction pad

L = 780 mm trajectory of the point in full revolution

H = 26,4 mm half thickness of the disk

Pe_0 = 24160 Péclet number for t=0

$$\Delta T \approx q \frac{2\sqrt{k}}{K\sqrt{\pi}} \sqrt{\Delta t} = \frac{2qa}{K\sqrt{\pi}\sqrt{\mathrm{Pe}}}$$

63,2°C

$$\bar{\sigma}_{xx} = \bar{\sigma}_{yy} \approx -\frac{2E\alpha\sqrt{k}}{(1-\nu)K\sqrt{\pi}} q\sqrt{\Delta t}$$

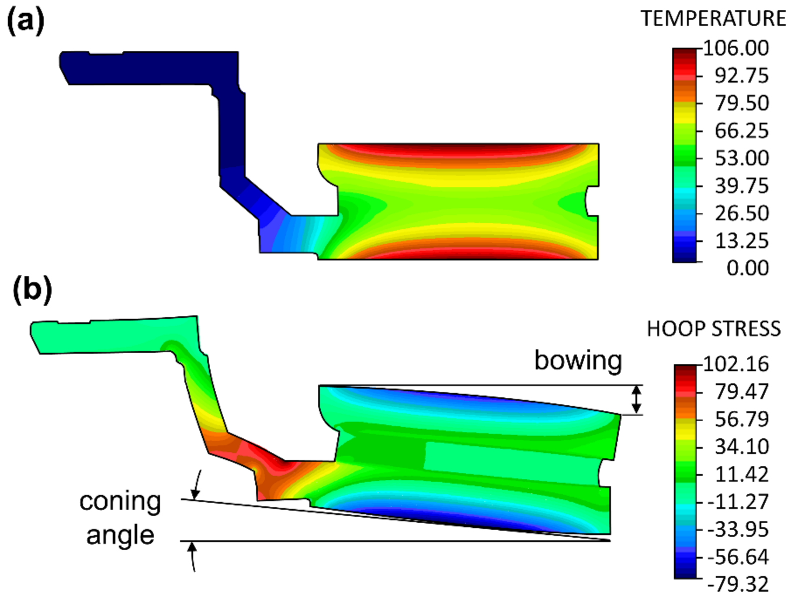
-126,4 MPa

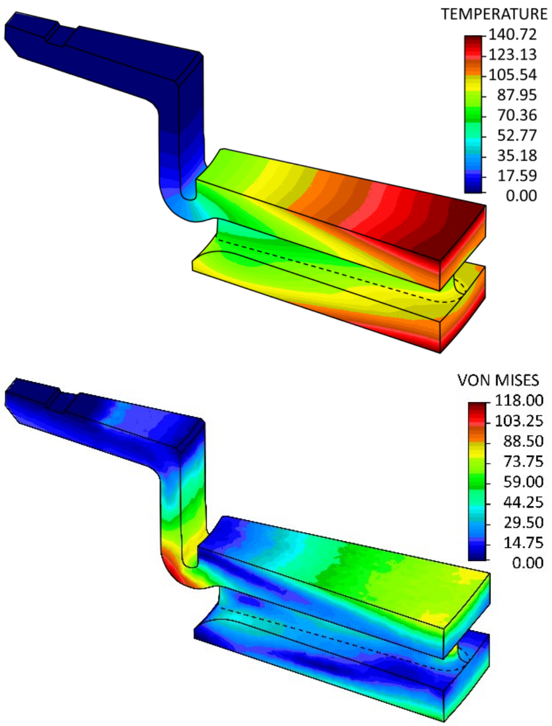
$$\Delta \bar{u}_z \approx -2c\, q\, a^2 / Pe$$

0.6 μm in the first revolution

Uncoupled thermal stress

Standard FEM models





Coupled thermal and distortion analysis

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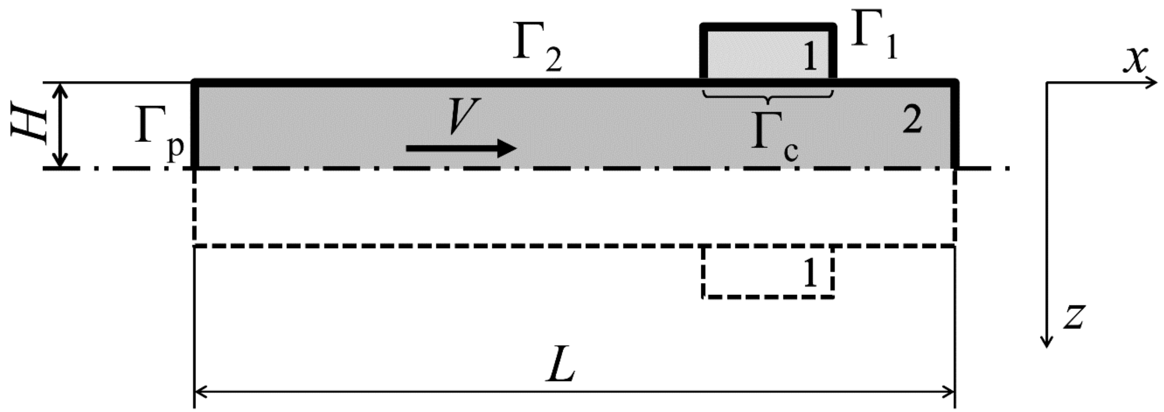
Coupled thermal analysis



$$c_i \rho_i \frac{\partial T_i}{\partial t} + V_i \frac{\partial T_i}{\partial x} = \frac{\partial}{\partial x}(K_i \frac{\partial T_i}{\partial x}) + \frac{\partial}{\partial z}(K_i \frac{\partial T_i}{\partial z}), \; i = 1,2,$$

where $V_2 = V(t)$, $V_1 = 0$

+ boundary conditions and temperature contact with the friction heating

The Galerkin FE discretization method is unstable, owing to the Pe > 2.

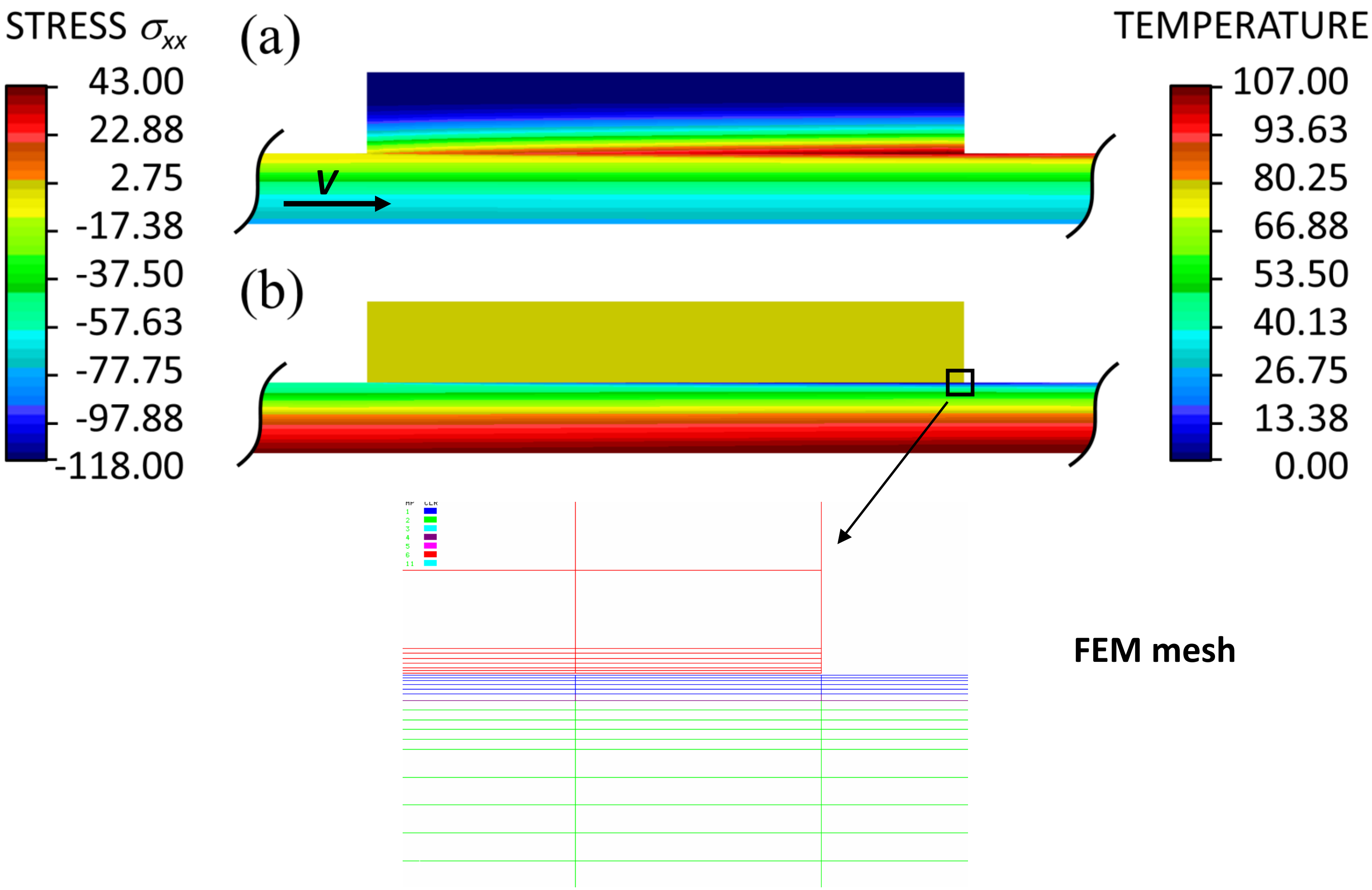
This difficulty can be removed using Petrov-Galerkin method.

An implementation of P-G method is, for example, in software ABAQUS.

Coupled thermal and distortion analysis

Example

Results obtained by my in-house software



Coupled thermal and distortion analysis

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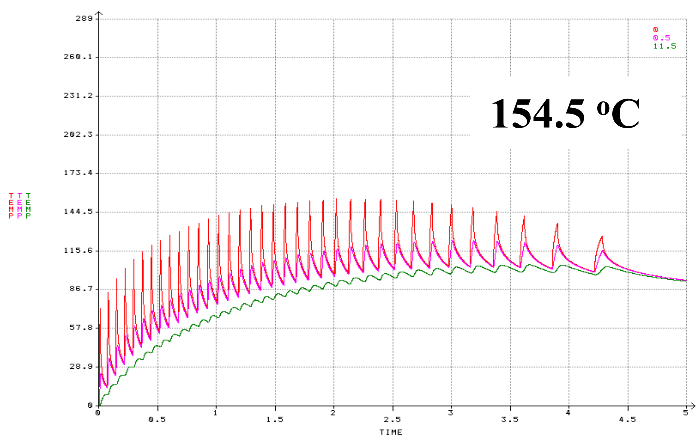
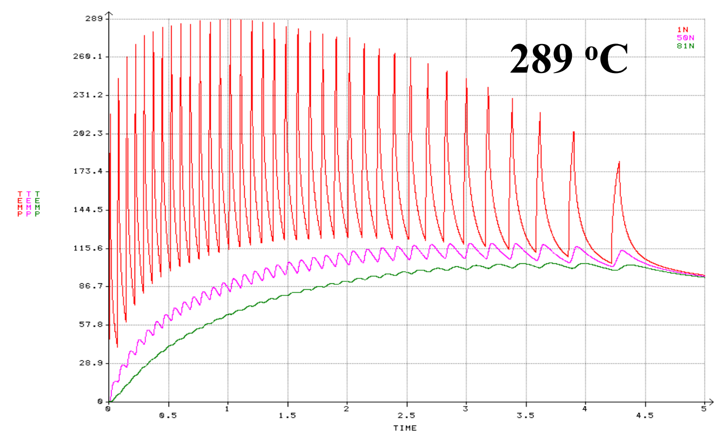
Example – A brake fading problem

Standard disk

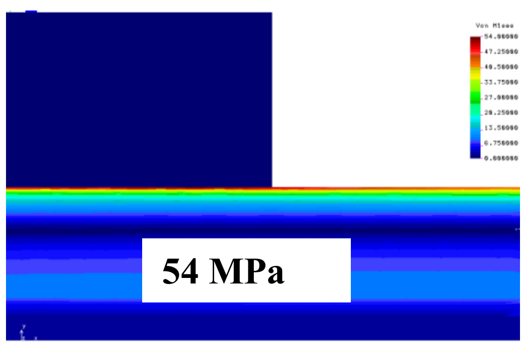
Disk with an inconvenient surface processing

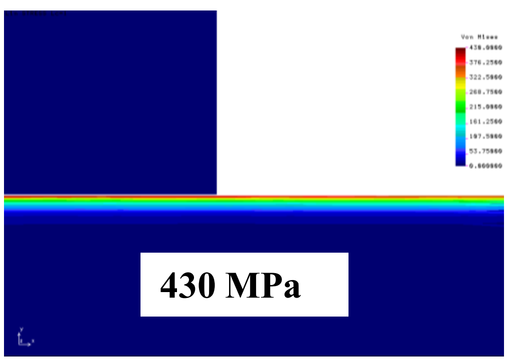
The flash temperature





The surface compressive stress



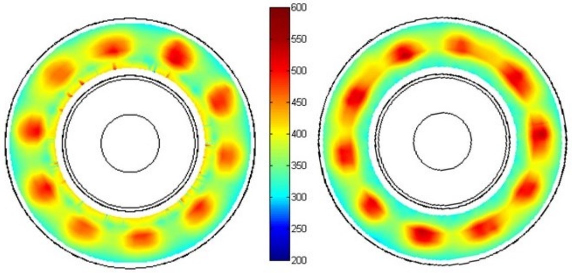


Coupled thermal and distortion analysis

Hot judder and hot spots

$$T_{max} = T_b + T_s + T_f + T_{per}$$

perturbace



Prof. J.R. Barber described the frictionally excited thermoelastic instability as cause of „hot judder“ a „hot spots“ at a high sliding velocity:

J.R. Barber: Thermoelastic instabilities in the sliding of conforming solids, *Proc. Roy. Soc.* A 312 (1969), 381-394.

the mutual coupling

thermal deformation – contact pressure – frictional heat generation – thermal deformation

Thank You very much