ABOUT ONE CLASS OF LINEAR TRANSFORMATIONS OF NON-STATIONARY RANDOM SEQUENCES

Cheremskaya N.V.
National Technical University «Kharkiv Polytechnic Institute», Kharkiv

The paper deals with the problem of linear transformations of non-stationary random sequences of certain classes.

Consider a random sequence $\xi(n,\omega)$ second order with $M\xi(n,\omega)=0$. Let $K(n,m)=M\xi(n,\omega)\overline{\xi(m,\omega)}$ is correlation function. After embedding $\xi(n,\omega)$ in Hilbert space H_ξ get the sequence x_n at H_ξ, where $K(n,m)=\langle x_n,x_m\rangle_{H_\xi}$. Sufficiently wide classes of random sequences within the framework of the correlation theory can be obtained by considering linear transformations of sequences. x_n at H_ξ. The sequence $z_n = Bx_n$ is called the dilation of the r-th order of the sequence x_n, if B is linear bounded operator in H_ξ and $\dim\overline{\left(I-B^*B\right)H_\xi}=r$. The function $W(n,m)=K(n+1,m)-K(n,m+1)$ of two discrete arguments will be called the correlation difference.

Theorem. In order to z_n be a first-order dilatation of a stationary sequence x_n, it is necessary and sufficient that it is correlation difference be:

$$W_{zz}(n,m)=W_{xx}(n-m)+\sum_{\alpha,\beta=1}^{2}\Phi_\alpha(n)J_{\alpha\beta}\overline{\Phi_\beta(m)},$$

where $W_{xx}(n-m)=2i\int_{-\pi}^{\pi}\sin\lambda e^{i(n-m)\lambda}dF(\lambda)$, $F(\lambda)-$ non-decreasing function of limited variation, $J_{\alpha\beta}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$, where $\Phi_\alpha(n)-$ linear functional from x_n and $\Phi_1(n+1)=\Phi_2(n)$.

Theorem. In order z_n to be a first-order dilation of a sequence $x_n = A^n x_0$, where A is a bounded dissipative operator with a discrete spectrum $\{\lambda_k\}$ and a one-dimensional non-Hermitian subspace, it is necessary and sufficient that it is correlation difference be: $W_{zz}(n,m)=i\varphi(n)\overline{\varphi(m)}+\sum_{\alpha,\beta=1}^{2}\Phi_\alpha(n)J_{\alpha\beta}\overline{\Phi_\beta(m)}$,

where $\varphi(n)=\sum_{k=1}^{\infty}C_k\Lambda_k(n)$; $\sum_{k=1}^{\infty}|C_K|^2<\infty$; $\Lambda_k(n)=-\overline{\frac{1}{2\pi i}\oint_\gamma \lambda^n\frac{\beta_k}{\overline{\lambda_k}-\lambda}\prod_{j=1}^{k-1}\frac{\lambda_j-\lambda}{\overline{\lambda_j}-\lambda}d\lambda}$;

$(J_{\alpha\beta})=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$, and $\Phi_\alpha(n)-$ linear functional from z_n.