OBTAINING GLOBAL SOLUTIONS OF SEMILUNAR DEGENERATE DIFFERENTIAL EQUATIONS FOR NONLINEAR ELECTRIC CIRCUITS

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Research results of the concrete mathematical models of radio engineering, control theory, hydrodynamics, economics, robotics technology testify that the class of models with differential-algebraic equations (DAEs) is important for practice. Such equations contain a degenerate operator at the derivative and they are also called degenerate, singular, algebraic-differential, descriptor. The Cauchy problem for the semilinear differential-algebraic equation (DAE)

$$\frac{d}{dt}[Ax(t)] + Bx(t) = f(t, x) \qquad (1)$$

with a nonlinear function on the right-hand side is considered in the paper. It is assumed that $t \geq 0$, $x \in \mathrm{R}^n$, $f(t,x) \in C\left([0,\infty) \times \mathrm{R}^n, \mathrm{R}^m\right)$, $A, B: \mathrm{R}^n \to \mathrm{R}^m$ are linear operators to which $m \times n$ matrices A, B correspond, the operator A is degenerate and the characteristic operator pencil $\lambda A + B$ is singular. In particular, underdetermined and overdetermined systems of the differential-algebraic equations correspond to this DAE.

The purpose of the paper is to find the conditions of existence and uniqueness of global in time solutions of the DAE. It is necessary to know about the existence of global solutions for the equation describing the real system model, because it guarantees a sufficiently long action term of this real system. The presence of the nonlinear function in the equation is one of the main problems for the investigation of global solvability. Another problem is a degenerate (singular) operator at the derivative. To obtain the main results the extending solution method in terms of differential inequalities with Lyapunov and La Salle functions, the method of spectral projectors and the special block representations of the singular pencil are used.

The basic result of the work is the theorem on the global solvability of the Cauchy problem for the semilinear DAE (1). In the case when the DAE corresponds to the overdetermined system of equations, the solution is unique. For the DAE corresponding to the underdetermined system of equations, the solution isn't unique and it depends on the functional parameter determining one of its components. It is important that the nonlinear function $f(t,x)$ may not satisfy the constraints of the global Lipchitz condition type. The rejection of such constraints is due to the fact that in many practical problems of radio engineering, electronics, mathematical economics the real nonlinearities aren't global Lipschitz. However, the systems of equations arising from the mathematical model construction of these problems may have global solutions. For example, the presence of cubic nonlinear resistances and conductivities in electric circuits, as a rule, admits the existence of global solutions.

The models of radio engineering filters with nonlinear elements are considered as applications. Check of the conditions of the proved theorem is carried out and it analysis have shown that the requirements of the theorem are physically feasible. The obtained numerical solutions of the considered DAEs verify the theoretical results.